EE538 DSPI

Module 8

Outline: DAC

- Sample and Hold (S/H) Reconstruction - Sect. 9.3.1
- Digital Up-Sampling - Sect. 10.3
- Efficient Upsampling - Sect. 10.5.2

- See Fig. 9.22 on pg. 767
  $\Rightarrow$ S/H reconstruction
- Digital Upsampling

$$x[n] \longrightarrow \boxed{\text{DT System}} \longrightarrow y[n]$$

$$= x_a\left(\frac{n}{F_s}\right) \qquad\qquad = x_a\left(\frac{n}{LF_s}\right)$$

- where L = integer
- i.e., increasing sampling by L just prior to S/H

- Preamble:

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

- in particular, if $n_0 = 0$:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

- Consider: $\displaystyle\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-2k]$

$$= \ldots + x[-1]\,\delta[n+2] + x[0]\,\delta[n] + x[1]\,\delta[n-2] + \ldots$$

$$= \{\ldots, 0, x[-1], 0, x[0], 0, x[1], 0, \ldots\}$$

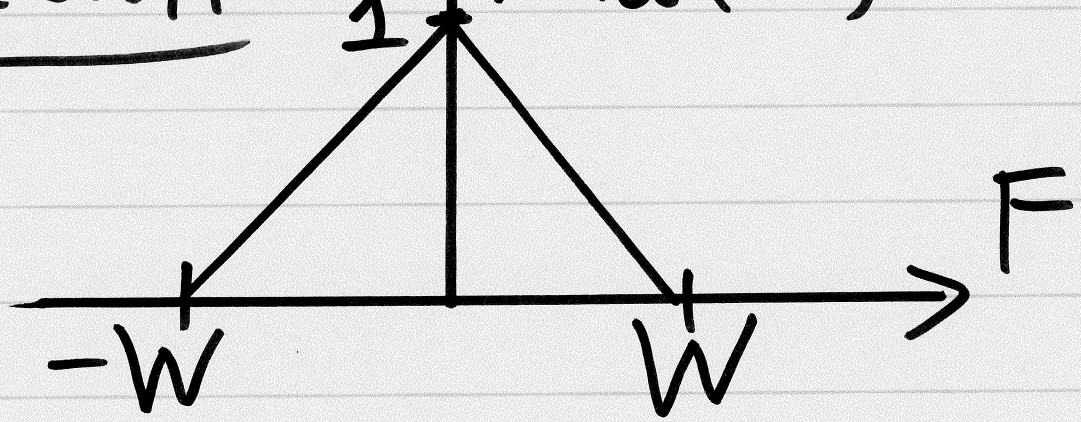
$$x[n] \longrightarrow \uparrow 2 \longrightarrow \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-2k]$$

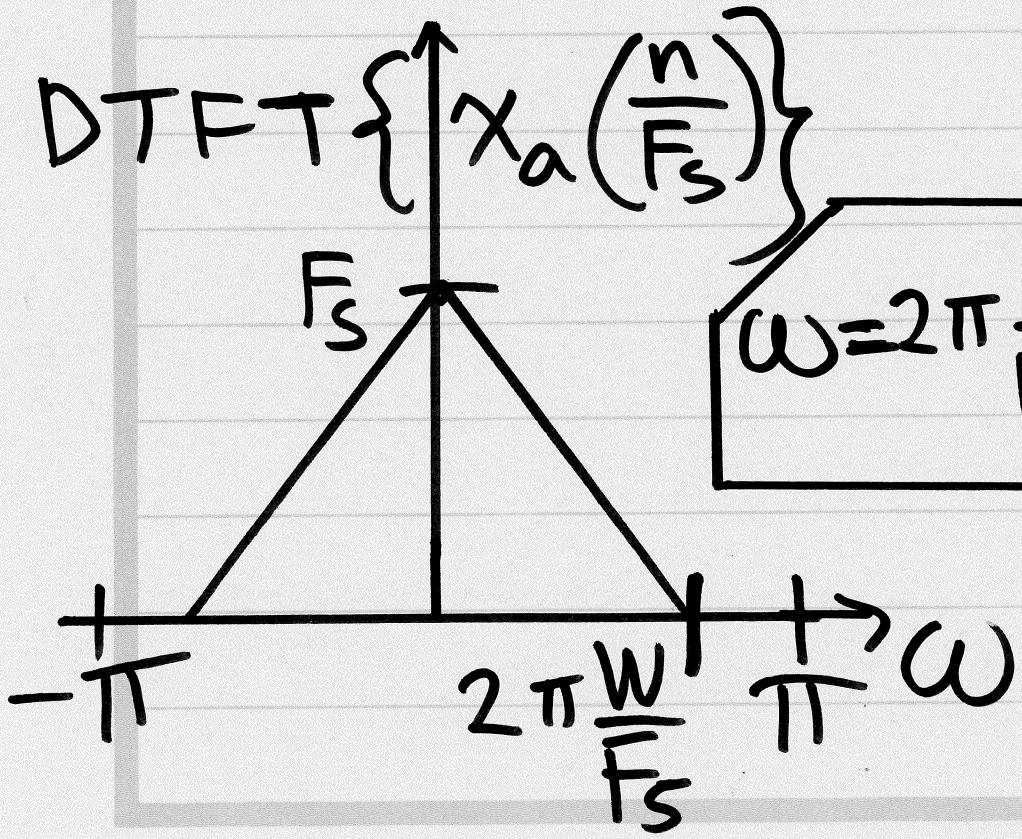
- more generally:
- insert $L-1$ zeroes between ea. pair of samples

$$x[n] \longrightarrow \uparrow L \longrightarrow \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL]$$

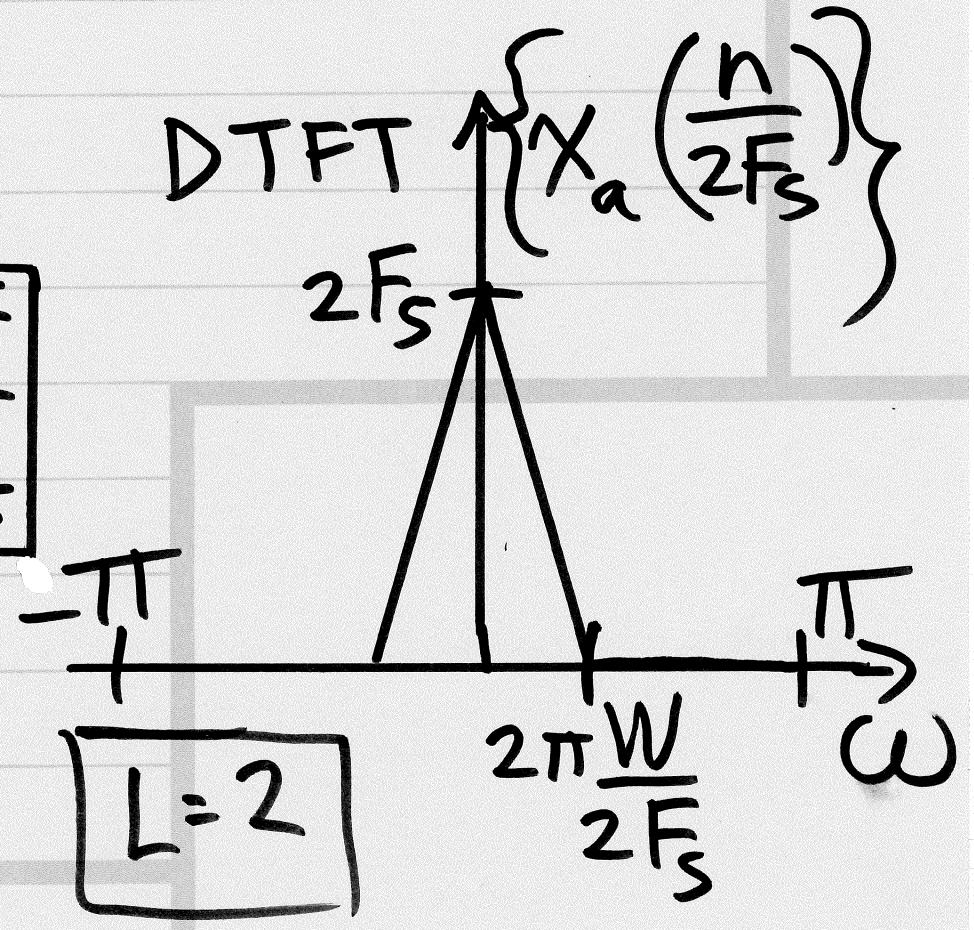
- End of Preamble

Recall: $X_a(F) = \mathcal{F}\{x_a(t)\}$





$$\boxed{\omega = 2\pi \frac{F}{F_s}}$$



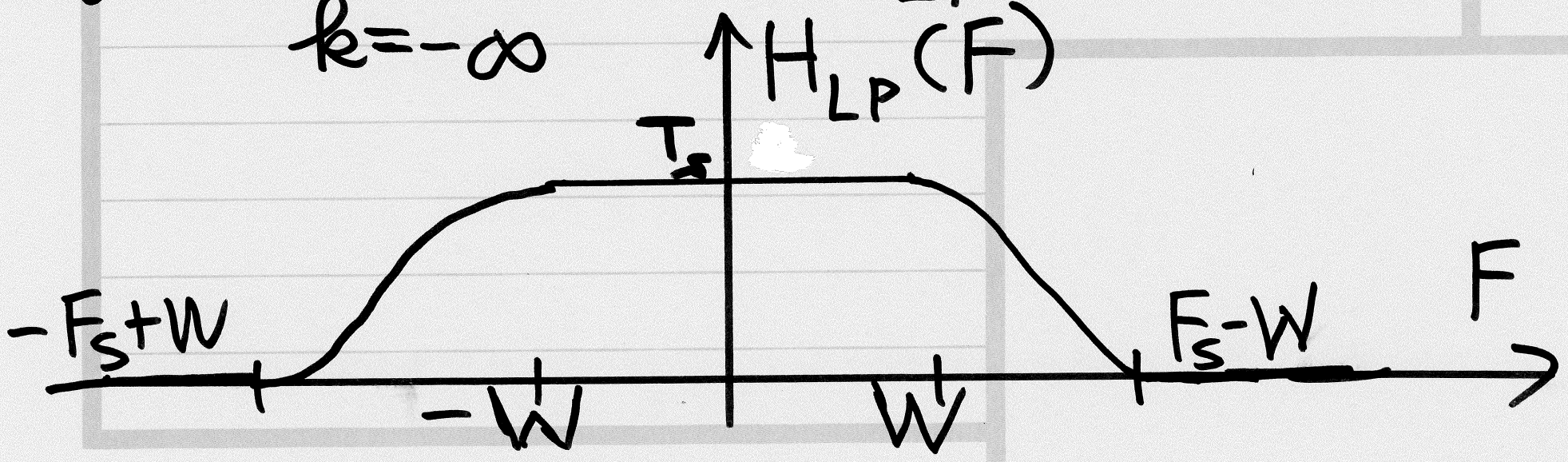
$$\boxed{L = 2}$$

- as long as $F_s > 2W$

(i.e., $|X_a(F)| \approx 0$ for $|F| > W$)

- then $x_a(t)$ may be reconstructed

$$x_a(t) = \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}(t - kT_s)$$



$$y[n] = x_a\left(n\frac{T_s}{L}\right) = x_a\left(\frac{n}{LF_s}\right)$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}\underbrace{\left(n\frac{T_s}{L} - kT_s\right)}_{(n-kL)\frac{T_s}{L}}$$

$$= \sum x[k]\, h_{LP}[n-kL]$$

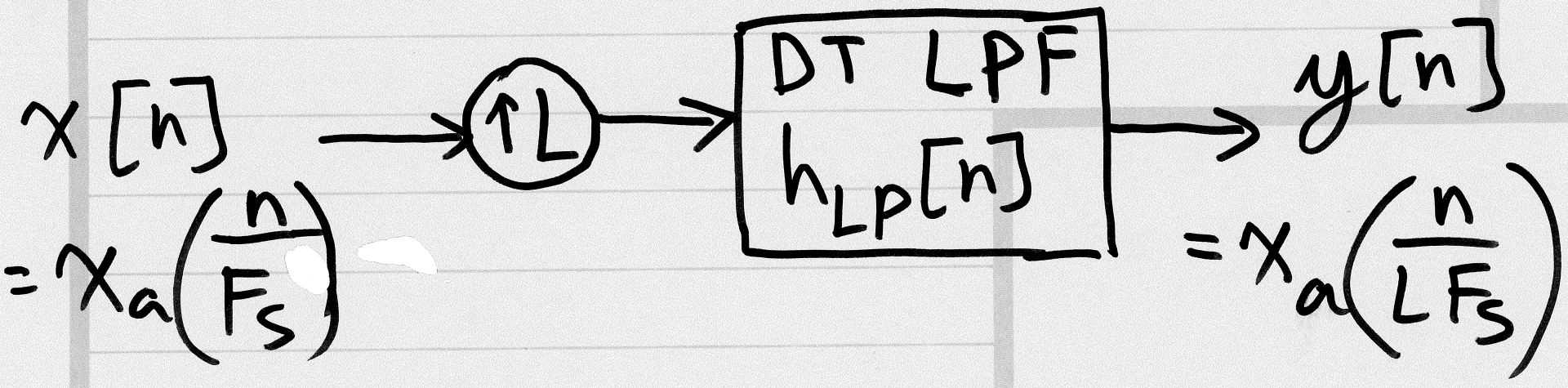
- where: $h_{LP}[n] = h_{LP}\left(n\frac{T_s}{L}\right)$

$$= h_{LP}\left(\frac{n}{LF_s}\right)$$

$$h_{LP}[n-kL] = h_{LP}[n] * \delta[n-kL]$$

$$y[n] = \left\{ \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL] \right\} * h_{LP}[n]$$

- convolution is distributive



- Ideal D/A Reconstruction achieved with:

$$h_{LP}(t) = T_s \frac{\sin\left(2\pi \frac{F_s}{2} t\right)}{\pi t} = T_s \frac{\sin\left(\frac{\pi}{T_s} t\right)}{\pi t}$$

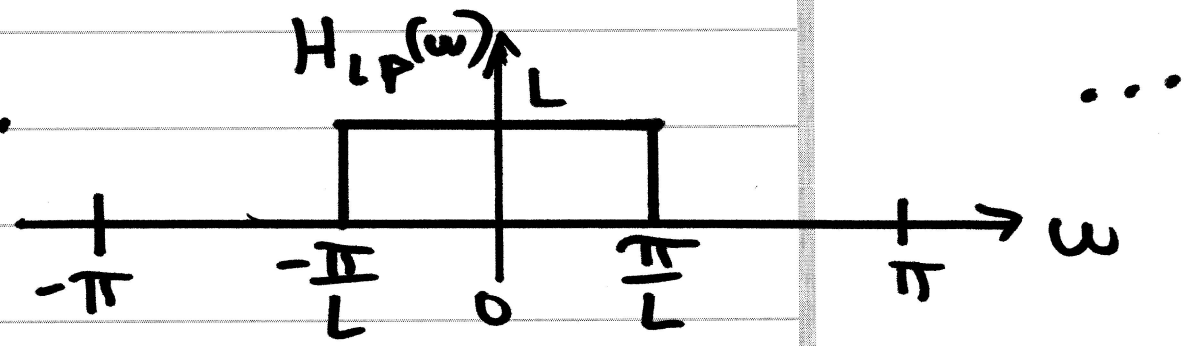
$$-\infty < t < \infty$$

- For digital upsampling by an integer factor of L

$$h_{LP}[n] = h_{LP}(t)\Big|_{t = n\frac{T_s}{L}}$$

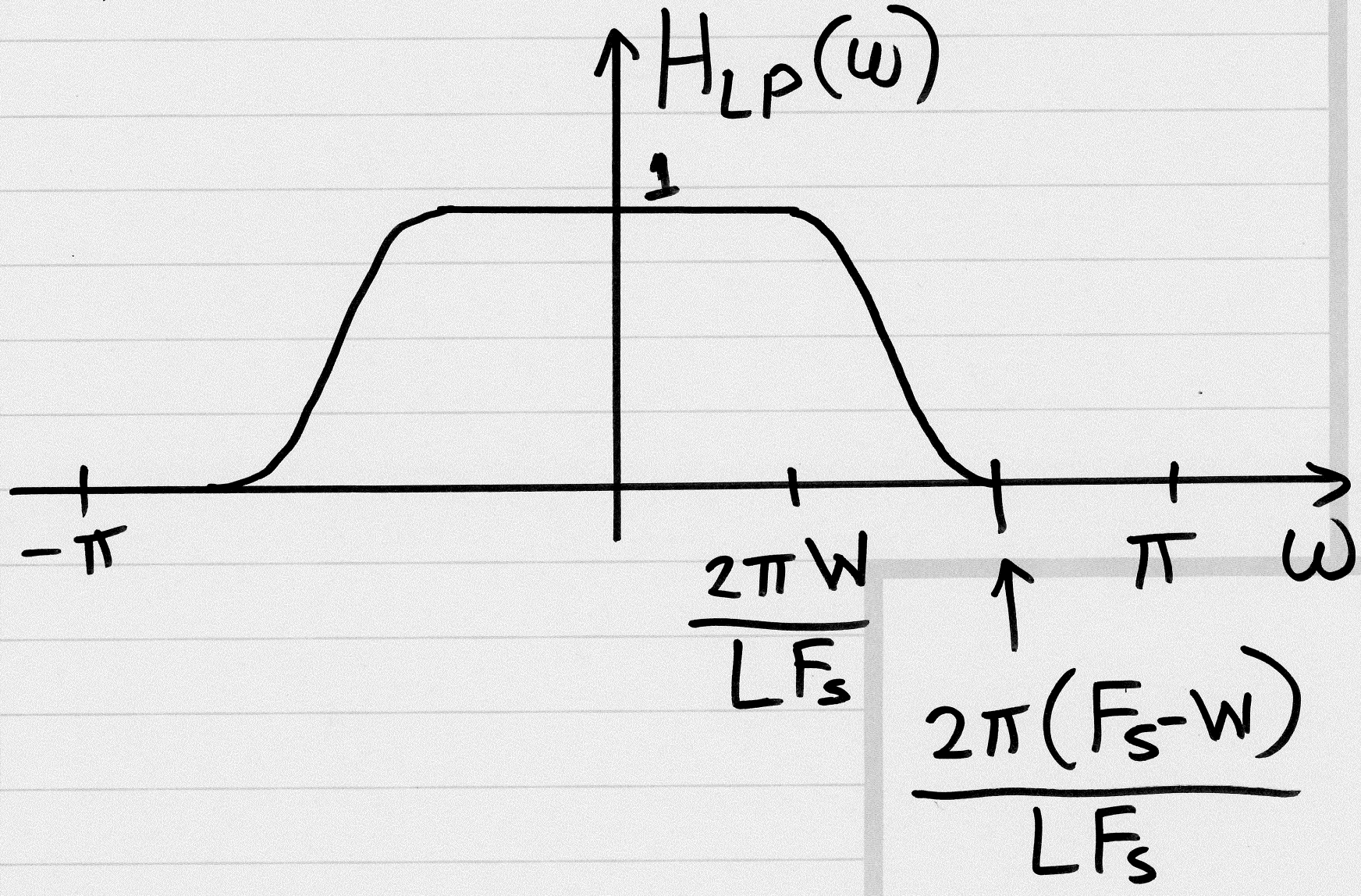
$$= \frac{T_s \sin\left(\frac{\pi}{T_s} n \frac{T_s}{L}\right)}{\pi n \frac{T_s}{L}} = \frac{L \sin\left(\frac{\pi}{L} n\right)}{\pi n} \qquad -\infty < n < \infty$$

$\xleftrightarrow{\text{DTFT}}$ ...



...

- Spec's on the digital LPF:



- <u>Example</u>: for speech with bandwidth $\approx$ 4 KHz sampled at $F_s = 12$ KHz
- desire to increase sampling rate by $L=2$ to 24 KHz
- $\omega_p = 2\pi \dfrac{4}{2(12)} = \dfrac{\pi}{3}$
- $\omega_s = 2\pi \dfrac{(12-4)}{2(12)} = \dfrac{2\pi}{3}$

See demo upsamplex2 eg1.m

- Efficient Digital Upsampling
- $L=2$ for illustration

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}[n-2k]$$

- consider $y[2n]$ and $y[2n+1]$ separately

$$y_o[n] = y[2n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}[\underbrace{2n-2k}_{2(n-k)}]$$

$$= x[n] * h_o[n]$$

- where: $h_o[n] = h_{LP}[2n]$

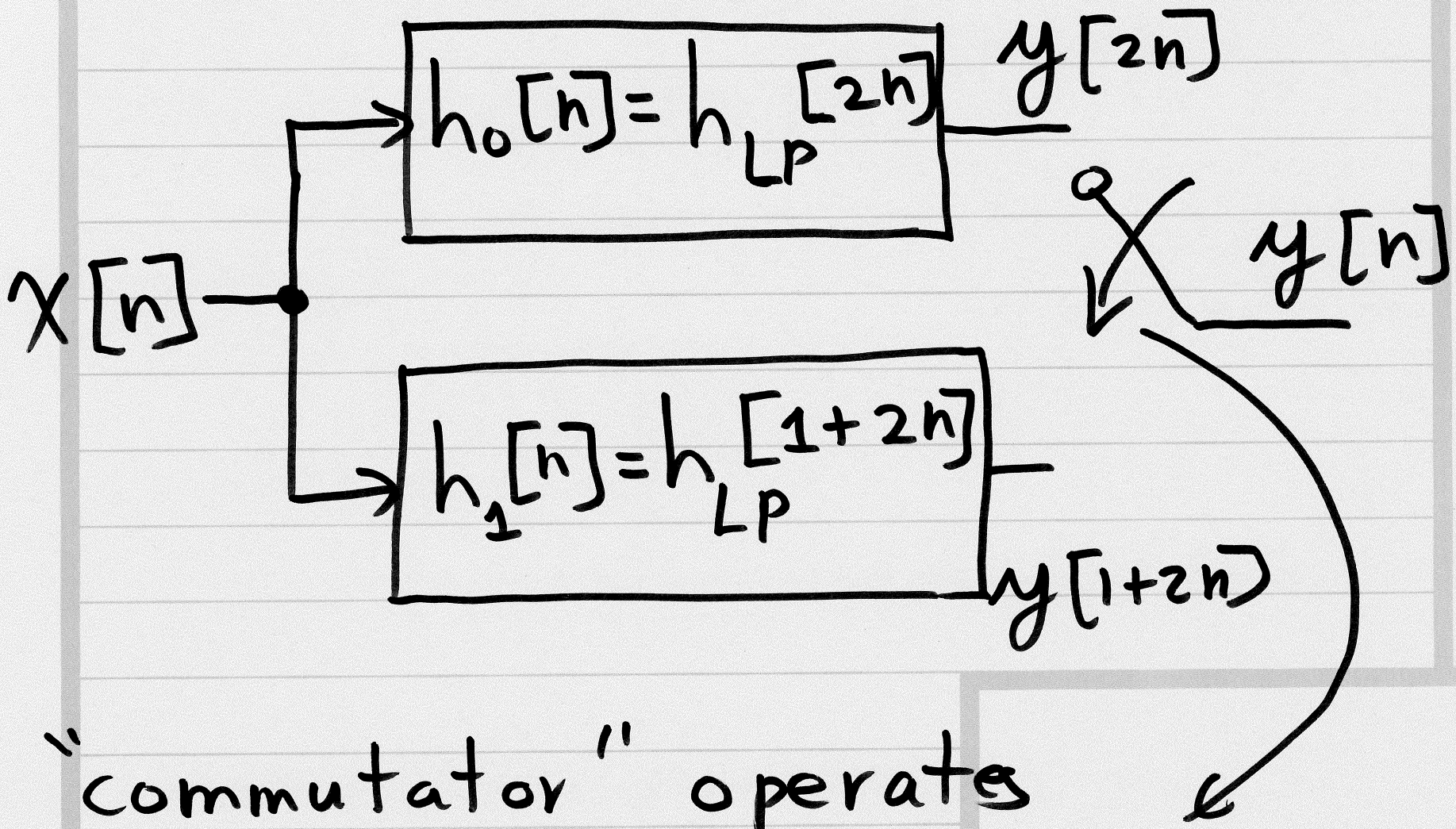
$$x[n] \longrightarrow \boxed{h_o[n] = h_{LP}[2n]} \longrightarrow y_o[n] = y[2n]$$

$$y_1[n] = y[1+2n]$$

$$= \sum x[k]\, h_{LP}\underbrace{[1+2n-2k]}_{1+2[n-k]}$$

$$= x[n] * h_1[n]$$

- where: $h_1[n] = h_{LP}[1+2n]$



"commutator" operates at twice original sampling rate

- efficient implementation for general case of up-sampling by L