EE538 DSP I

Module 7

Outline:

- Aliasing examples - see handout
- Sampling Theorem - Ideal Reconstruction Formula - Sect. 4.2.9
- DAC - Digital-to-Analog Conversion - Sect. 9.3

- Derive ideal reconstruction formula
- if $F_s > 2W$, then

$$X(\omega) = F_s X_a\left(\frac{F_s}{2\pi}\omega\right) \quad \text{for } |\omega| < \pi$$

- in turn:

$$X_a(F) = \frac{1}{F_s} X\left(2\pi \frac{F}{F_s}\right) \qquad \omega = 2\pi \frac{F}{F_s}$$

$$\text{for: } -\frac{F_s}{2} < F < \frac{F_s}{2}$$

$$X_a(F) = \frac{1}{F_s}\sum_{n=-\infty}^{\infty} x[n]\, e^{-j 2\pi \frac{F}{F_s} n} \quad \text{for } |F| < \frac{F_s}{2}$$

$$x_a(t) = \int_{-\infty}^{\infty} X_a(F)\, e^{+j 2\pi F t}\, dF$$

$$x_a(t) = \int_{-F_s/2}^{F_s/2} \frac{1}{F_s}\sum_{n=-\infty}^{\infty} x[n]\, e^{-j 2\pi \frac{F}{F_s} n}\, e^{j 2\pi F t}\, dF$$

$$x_a(t) = T_s \sum_n x[n] \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} e^{j2\pi F(t-nT_s)}\, dF$$

$$= T_s \sum_n x[n] \frac{1}{j2\pi(t-nT_s)} e^{j2\pi F(t-nT_s)} \Big|_{-\frac{F_s}{2}}^{\frac{F_s}{2}}$$

$$= \sum x[n] \frac{\sin\left[\frac{\pi}{T_s}(t-nT_s)\right]}{\frac{\pi}{T_s}(t-nT_s)}$$

See pp. 274-275

- Diversion: Dirac Delta Fn.

$$\delta_a(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} \operatorname{rect}\left(\frac{t}{\Delta}\right)$$

- Some properties: $\delta_a(t) = 0$ for $t \neq 0$

$$\int_{t_0-\epsilon}^{t_0+\epsilon} \delta_a(t-t_0)\,dt = 1$$

- Sifting property:

$$\int_{-\infty}^{\infty} x_a(t)\,\delta_a(t-t_0)\,dt = x_a(t_0)$$

• Convolution Prop.:

$$x_a(t) * \delta_a(t-t_0)$$

$$= \int_{-\infty}^{\infty} x_a(t-\lambda)\,\delta_a(\lambda-t_0)\,d\lambda$$

$$= \int_{-\infty}^{\infty} x_a(t-t_0)\,\delta_a(\lambda-t_0)\,d\lambda$$

$$= x_a(t-t_0)$$

- $\delta_a(t-t_0) \xleftrightarrow{\text{CTFT}} \ ?$

$$\int_{-\infty}^{\infty} \delta_a(t-t_0)\, e^{-j2\pi F t}\, dt$$

$$= e^{-j2\pi F t_0} \qquad -\infty < F < \infty$$

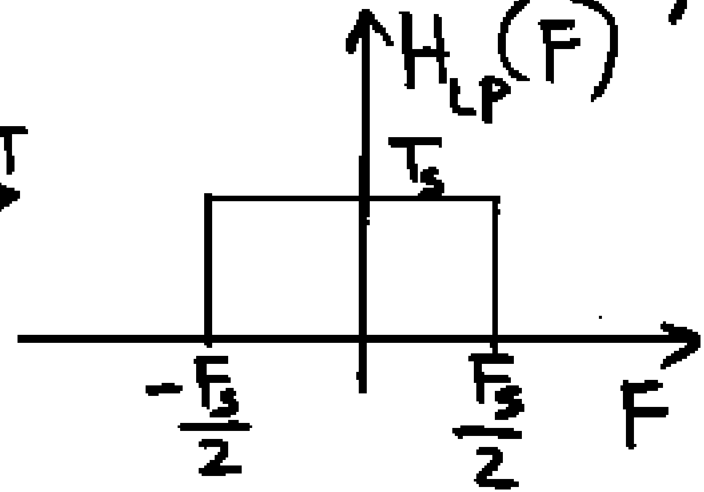
- End of Diversion

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT_s) \underbrace{\frac{\sin\left[\frac{\pi}{T_s}(t-nT_s)\right]}{\frac{\pi}{T_s}(t-nT_s)}}$$

$$\Rightarrow \frac{\sin\left(\frac{\pi}{T_s}t\right)}{\frac{\pi}{T_s}t} * \delta_a(t-nT_s)$$

$$x_a(t) = \left\{\sum_n x_a(nT_s)\,\delta_a(t-nT_s)\right\} * \frac{\sin\left(\frac{\pi}{T_s}t\right)}{\frac{\pi}{T_s}t}$$

$$\underbrace{h_{LP}(t) = \frac{\sin\left(\frac{\pi}{T_s}t\right)}{\frac{\pi}{T_s}t}}_{\text{impulse response of Ideal Lowpass Filter (ILPF)}} \xleftrightarrow{\text{CTFT}}$$



impulse response of Ideal Lowpass Filter (ILPF)

$$\sum_n x_a(nT_s)\,\delta_a(t-nT_s) = x_s(t) \longrightarrow \boxed{\begin{array}{l}\text{ILPF} \\ \text{cut-off} = \frac{F_s}{2}\end{array}} \longrightarrow x_a(t)$$

$$x_s(t) = \sum_n x_a(nT_s)\,\delta_a(t - nT_s) \xleftrightarrow{\text{CTFT}} X_s(F) = ?$$

$$X_s(F) = \sum_n x_a(nT_s)\,\mathcal{F}\{\delta_a(t-nT_s)\}$$

$$= \sum_{n=-\infty}^{\infty} x_a(nT_s)\, e^{-j2\pi F n T_s}$$

$$\omega = 2\pi \frac{F}{F_s}$$

- compare with DTFT:

$$X(\omega) = \sum_n x[n]\, e^{-j\omega n}$$

$$\underbrace{X_s(F)}_{} = X(\omega)\Big|_{\omega = 2\pi \frac{F}{F_s}}$$

periodic with period $= F_s$