EE538 DSP I

Module 5

Outline:

- Effects of Pole & Zero Locations on Frequency Response • Sect. 5.2.2
  - see zpgui.m at course web site
- Discrete-Time Fourier Transform (DTFT)
- Digital Notch Filters - Sect. 5.4.4

- Frequency analysis of DT Signals
- arbitrary (stable) $x[n]$ may be represented as:

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega \quad \Big\}\ \text{inverse DTFT}$$

- where $X(\omega)$ is the DTFT of $x[n]$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = X(z)\Big|_{z=e^{j\omega}}$$

- note: $e^{j(\omega+2\pi)n} = e^{j\omega n}\cdot e^{j2\pi n} = e^{j\omega n}$

- Riemann integration dictates:

$$x[n] = \lim_{N\to\infty} \sum_{k=-N}^{N} \frac{1}{N} X\left(\frac{k\pi}{N}\right) e^{j\frac{k\pi}{N}n}$$

$$\underbrace{\Delta\omega = \frac{2\pi}{N}}_{\text{width}} \times \underbrace{\frac{1}{2\pi} X\left(\frac{k\pi}{N}\right) e^{j\frac{k\pi}{N}n}}_{\text{height}}$$

of N rectangles
equi-spaced over $-\pi < \omega < \pi$

- $x[n]$ is an (infinite) sum of (complex) sinewaves infinitesimally spaced in frequency

- recall: $x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$$\Rightarrow \quad Y(z) = H(z)\,X(z)$$

- on unit circle:

$$Y(\omega) = H(\omega)\,X(\omega)$$

$$x[n] * y[n] \xleftrightarrow{\text{DTFT}} H(\omega)\,X(\omega)$$

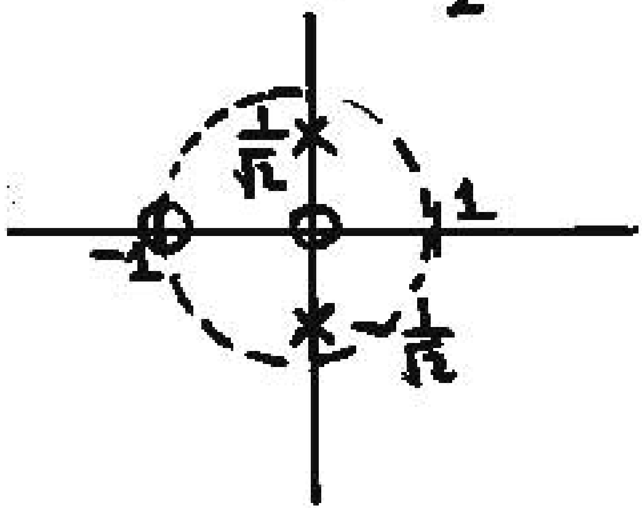
- through judicious positioning of zeroes and poles, can
- emphasize "desired" frequency bands
- de-emphasize other frequency bands
  - See Fig. 4.43 on pg. 331
  - See Fig. 4.44 on pg. 334

For 4th Ed, see Fig. 5.4.1 on pg. 327
Also see Fig. 5.4.2 on pg. 330.

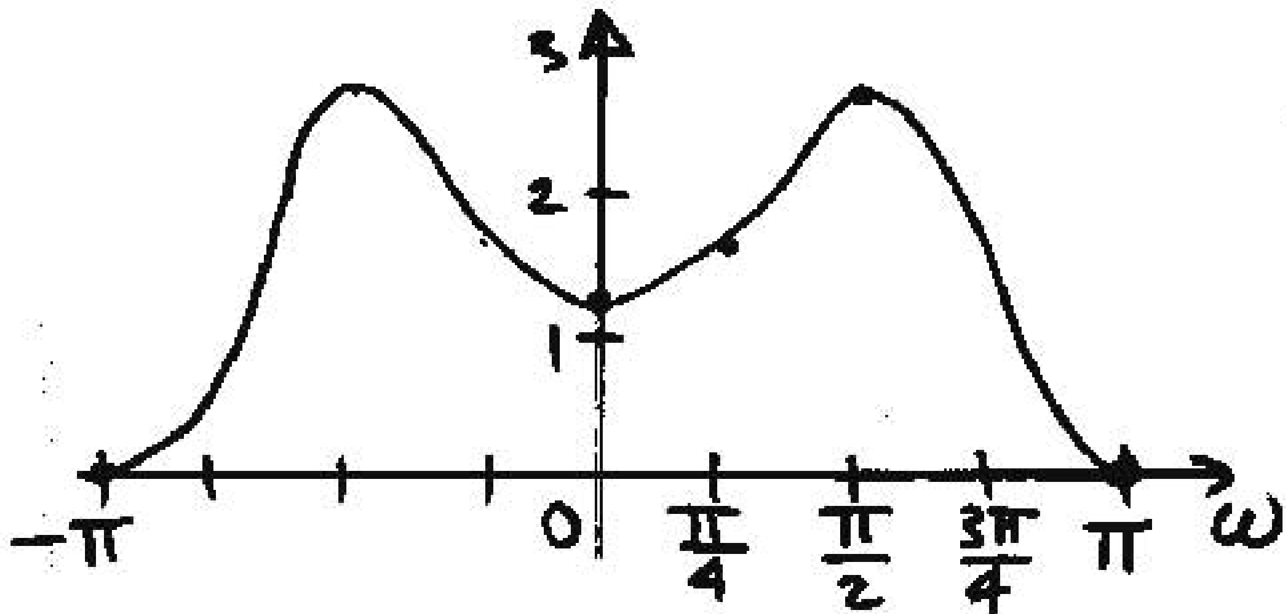
- **Example**

$$y[n] = -\frac{1}{2} y[n-2] + x[n] + x[n-1]$$

$$H(z) = \frac{1 + z^{-1}}{1 + \frac{1}{2} z^{-2}} = \frac{z(z+1)}{\left(z - j\frac{1}{\sqrt{2}}\right)\left(z + j\frac{1}{\sqrt{2}}\right)}$$

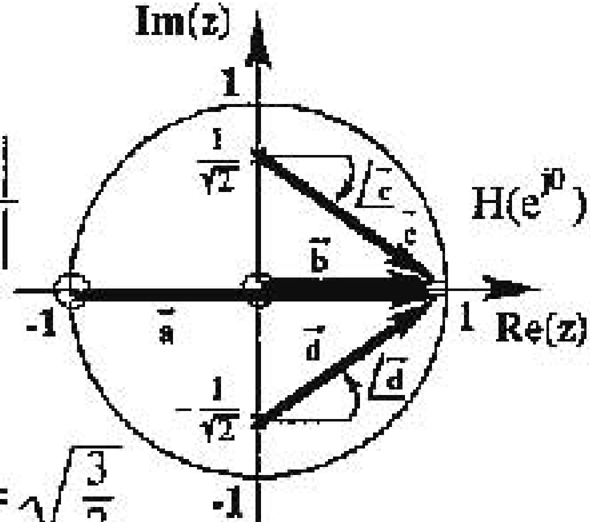


- See handout for graphical evaluation of $|H(\omega)|$ and $\angle H(\omega)$



- End of Example

C $\underline{\omega = 0}$



$$\left|H(e^{j0})\right| = \frac{|\vec{a}||\vec{b}|}{|\vec{c}||\vec{d}|}$$

$$|\vec{a}| = 2$$

$$|\vec{b}| = 1$$

$$|\vec{c}| = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$|\vec{d}| = \sqrt{\frac{3}{2}}$$

$$\left|H(e^{j0})\right| = \frac{4}{3} = 1.33$$

$$\angle H(e^{j0}) = \angle\vec{a} + \angle\vec{b} - \angle\vec{c} - \angle\vec{d}$$

$$\angle\vec{a} = 0$$

$$\angle\vec{b} = 0$$

$$\angle\vec{c} = \arctan\left(\frac{1}{\sqrt{2}}\right)$$

$$\angle\vec{d} = -\angle\vec{c}$$

$$\angle H(e^{j0}) = 0$$

$\underline{\omega = \pi / 4}$

$$\left|H(e^{j\pi/4})\right| = \frac{|\vec{a}||\vec{b}|}{|\vec{c}||\vec{d}|}$$

$$|\vec{a}| = \sqrt{\left(1 + \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}}$$

$$= 1.85$$

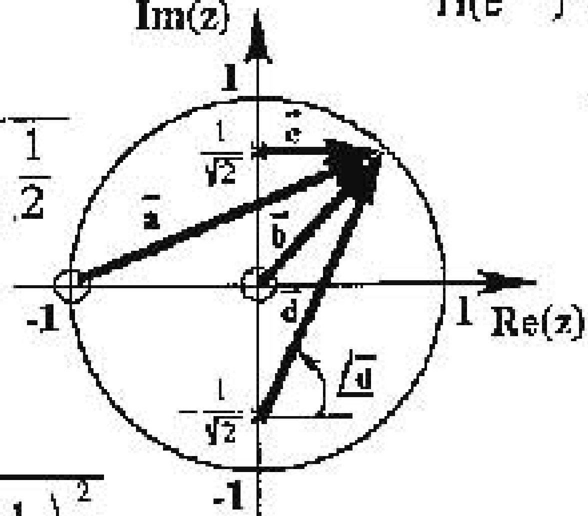
$$|\vec{b}| = 1$$

$$|\vec{c}| = \frac{1}{\sqrt{2}}$$

$$|\vec{d}| = \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{5}{2}}$$

$$\left|H(e^{j\pi/4})\right| = 1.65$$



$$\angle H(e^{\pi/4}) = \angle\vec{a} + \angle\vec{b} - \angle\vec{c} - \angle\vec{d}$$

$$\angle\vec{a} = 22.5°$$

$$\angle\vec{b} = 45°$$

$$\angle\vec{c} = 0$$

$$\angle\vec{d} = 63.4°$$

$$\angle H(e^{j\pi/4}) = 4.$$

$\underline{\omega = \pi/2}$

$$\left|H(e^{j\pi/2})\right| = \frac{|\vec{a}||\vec{b}|}{|\vec{c}||\vec{d}|}$$

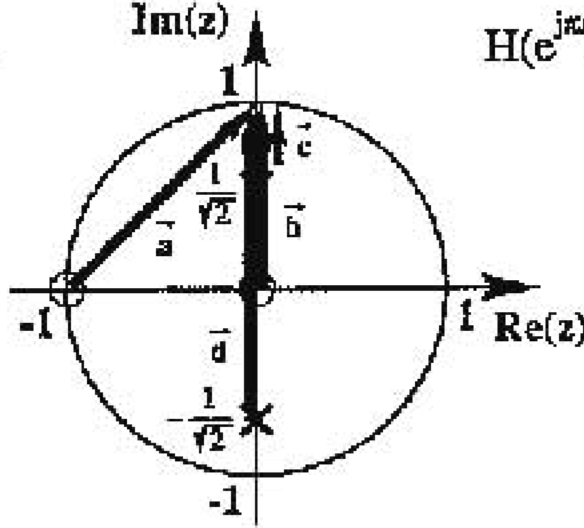
$$|\vec{a}| = \sqrt{2}$$

$$|\vec{b}| = 1$$

$$|\vec{c}| = 1 - \frac{1}{\sqrt{2}}$$

$$|\vec{d}| = 1 + \frac{1}{\sqrt{2}}$$

$$\left|H(e^{j\pi/2})\right| = 2.83$$



$$\angle H(e^{j\pi/2}) = \angle\vec{a} + \angle\vec{b} - \angle\vec{c} - \angle\vec{d}$$

$$\angle\vec{a} = 45°$$

$$\angle\vec{b} = 90°$$

$$\angle\vec{c} = 90°$$

$$\angle\vec{d} = 90°$$

$$\angle H(e^{j\pi/2}) = -45$$

• Notch Filters

$$H_{notch}(z) = \frac{G\,(z-e^{j\omega_0})(z-e^{-j\omega_0})}{(z-re^{j\omega_0})(z-re^{-j\omega_0})}$$

• where ω_0 is the frequency to be notched

$$\rightarrow \boxed{H_{FIR}(z)} \rightarrow \boxed{H_{IIR}(z)} \rightarrow$$

$$(z-e^{j\omega_0})(z-e^{-j\omega_0}) \qquad \frac{G}{(z-re^{j\omega_0})(z-re^{-j\omega_0})}$$

See Fig. 4.5.1 and 4.5.2 in Text

- ultimately the difference eqn. that's implemented is:

$$y[n] = 2r\cos(\omega_0)\,y[n-1] - r^2 y[n-2] + G\,x[n] + G\,2\cos(\omega_0)\,x[n-1] + G\,x[n-2]$$