

EE538

DSP I

Module 4

Outline:

- Z-Transform (cont.)
- Sects. 3.4.3 and Sect. 3.6.4
- Relationship between ZT and DTFT
- Sect. 4.2.6, 4.4.2

• Onto Chap. 4 on DTFT 12  
Discrete-Time Fourier Transform

• Recall:

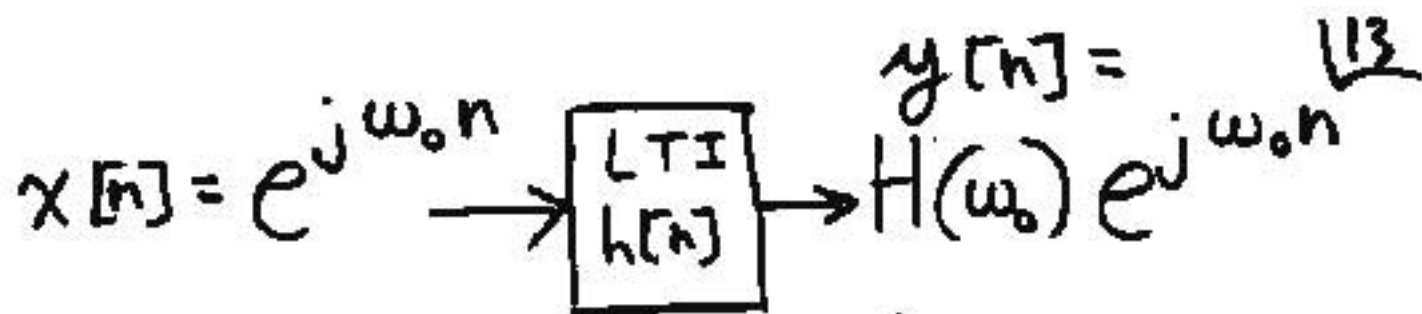


• Consider  $z_0$  on the unit circle

• recall:  $e^{j\theta} = \cos \theta + j \sin \theta$

•  $z_0 = e^{j0} = 1$  ;  $z_0 = e^{j\pi} = -1 = e^{-j\pi}$

•  $z_0 = e^{j\frac{\pi}{2}} = j$  ;  $z_0 = e^{-j\frac{\pi}{2}} = -j$



• where:  $H(\omega) = H(z) \Big|_{z=e^{j\omega}}$

notational problem

•  $H(\omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$

} DTFT  
 of  $h[n]$

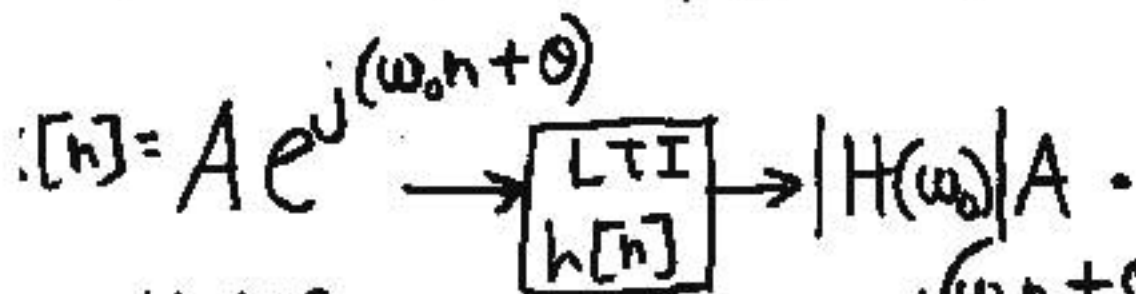
• only defined if ROC includes  $|z|=1$

$\Rightarrow$  only defined for stable systems

$$H(\omega_0) = |H(\omega_0)| e^{j\angle H(\omega_0)}$$

(14)

$$x[n] = A e^{j(\omega_0 n + \theta)}$$



$$-\infty < n < \infty$$

$$\times e^{j(\omega_0 n + \theta + \angle H(\omega_0))}$$

• recall:

$$\cos(\omega_0 n) = \frac{1}{2} e^{j\omega_0 n} + \frac{1}{2} e^{-j\omega_0 n}$$

$$A \cos(\omega_0 n + \theta) \rightarrow \boxed{h[n]} \rightarrow A |H(\omega_0)| \cdot$$

$$\times \cos(\omega_0 n + \theta + \angle H(\omega_0))$$

• Return to Difference Eqns. 15

$$y[n] = -\sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$$
$$= b_0 z^{N-M} \frac{\prod_{k=1}^M (z - z_k)}{\prod_{k=1}^N (z - p_k)}$$

$$H(\omega) = H(z) \Big|_{z=e^{j\omega}}$$

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$$H(\omega) = b_0 e^{j\omega(N-M)} \frac{\prod_{k=1}^M (e^{j\omega} - z_k)}{\prod_{k=1}^N (e^{j\omega} - p_k)}$$

$$|H(\omega)| = |b_0| \frac{\prod_{k=1}^M |e^{j\omega} - z_k|}{\prod_{k=1}^N |e^{j\omega} - p_k|}$$

$$\angle H(\omega) = \omega(N-M) + \angle b_0 \quad \boxed{17}$$
$$+ \sum_{k=1}^M \angle(e^{j\omega} - z_k) - \sum_{k=1}^N \angle(e^{j\omega} - p_k)$$

• See pg. 323 in  
Text for vector  
interpretation