EE538 DSP I

Module 4

Outline:

- Z-Transform (cont.)
  - Sects. 3.4.3 and Sect. 3.6.4
- Relationship between ZT and DTFT

Sect. 4.2.6, 4.4.2

- Onto Chap. 4 on DTFT

Discrete-Time Fourier Transform

- Recall:

$$x[n] = z_0^n \longrightarrow \boxed{\begin{matrix} \text{LTI} \\ h[n] \end{matrix}} \longrightarrow y[n] = H(z_0)\, z_0^n$$

$\forall n$

- Consider z_0 on the unit circle
- recall: $e^{j\theta} = \cos\theta + j\sin\theta$
- $z_0 = e^{j0} = 1$; $z_0 = e^{j\pi} = -1 = e^{-j\pi}$
- $z_0 = e^{j\frac{\pi}{2}} = j$; $z_0 = e^{-j\frac{\pi}{2}} = -j$

$$x[n] = e^{j\omega_0 n} \xrightarrow{\begin{array}{c}\text{LTI} \\ h[n]\end{array}} y[n] = H(\omega_0)\, e^{j\omega_0 n}$$

- where : $H(\omega) \underbrace{=}_{\text{notational problem}} H(z)\Big|_{z = e^{j\omega}}$

- $H(\omega) = \displaystyle\sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n}$ $\Big\}$ DTFT of $h[n]$

- only defined if ROC includes $|z| = 1$

  $\Rightarrow$ only defined for stable systems

$$H(\omega_0) = |H(\omega_0)|\, e^{j\angle H(\omega_0)}$$

$$x[n] = A e^{j(\omega_0 n + \theta)} \longrightarrow \boxed{\begin{matrix} \text{LTI} \\ h[n] \end{matrix}} \longrightarrow |H(\omega_0)| A \cdot e^{j(\omega_0 n + \theta + \angle H(\omega_0))}$$

$-\infty < n < \infty$

- recall:

$$\cos(\omega_0 n) = \frac{1}{2} e^{j\omega_0 n} + \frac{1}{2} e^{-j\omega_0 n}$$

$$A\cos(\omega_0 n + \theta) \longrightarrow \boxed{h[n]} \longrightarrow A\,|H(\omega_0)| \cdot \cos(\omega_0 n + \theta + \angle H(\omega_0))$$

- Return to Difference Eqns.

$$y[n] = -\sum_{k=1}^{N} a_k\, y[n-k] + \sum_{k=0}^{M} b_k\, x[n-k]$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$= b_0 z^{N-M} \frac{\prod_{k=1}^{M} (z - z_k)}{\prod_{k=1}^{N} (z - p_k)}$$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}}$$

$$H(\omega) = b_0 e^{j\omega(N-M)} \frac{\prod_{k=1}^{M}\left(e^{j\omega} - z_k\right)}{\prod_{k=1}^{N}\left(e^{j\omega} - p_k\right)}$$

$$|H(\omega)| = |b_0| \prod_{k=1}^{M}\left|e^{j\omega} - z_k\right| \Big/ \prod_{k=1}^{N}\left|e^{j\omega} - p_k\right|$$

$$\angle H(\omega) = \omega(N-M) + \angle b_0 + \sum_{k=1}^{M} \angle(e^{j\omega} - z_k) - \sum_{k=1}^{N} \angle(e^{j\omega} - p_k)$$

- See pg. 323 in Text for vector interpretation