EE538 DSPI

Module 26

Outline:

- AR Spectral Estimation
  - Sect. 11.1.1 & 11.1.2
- Matlab demos: ARspecest.m, ARMAestviaAR.m, SOSestviaAR.m
- Relationship AR spectral estimation and linear prediction - Sect. 11.2.1

- Prove that for $x[n]$ a $AR(p)$ process, its autocorrelation sequence $r_{xx}[m] = E\{x[n]x^*[n-m]\}$ satisfies:

$$r_{xx}[m] = -\sum_{k=1}^{p} a_k\, r_{xx}[m-k] + \sigma_w^2 \delta[m] \qquad \text{for } m \geq 0$$

$$E\left\{\left[-\sum_{k=1}^{p} a_k\, x[n-k] + v[n]\right] x^*[n-m]\right\}$$

$$r_{xx}[m] = -\sum_{k=1}^{P} a_k\, r_{xx}[m-k] + E\{v[n]x^*[n-m]\}$$

- for $m \geq 0$ (since $r_{xx}[-m] = r_{xx}^*[m]$)

$$x^*[n-m] = -\sum_{k=1}^{P} a_k^*\, x^*[n-m-k] + v^*[n-m]$$

- only $v[n-m], v[n-m-1], \ldots, v[n-m-P]$ contribute to $x[n-m]$
- since $E\{v[n]v^*[n-m]\} = \sigma_w^2 \delta[m]$
- then $E\{v[n]x^*[n-m]\} = 0$ for $m > 0$

• for $m=0$: $E\{v[n]x^*[n]\} =$

$$= -E\left\{\sum_{k=1}^{p} a_k^* v[n]x^*[n-k]\right\} + E\{v[n]v^*[n]\}$$

$$= -\sum_{k=1}^{p} a_k^* \underbrace{E\{v[n]x^*[n-k]\}}_{\text{just proved}=0} + \sigma_w^2$$

• Thus, for $m \geq 0$:

$$r_{xx}[m] = -\sum_{k=1}^{p} a_k\, r_{xx}[m-k] + \sigma_w^2 \delta[m]$$

Q.E.D.

• Given $r_{xx}[m]$, $m = 0, 1, \ldots, P$, a_k, $k = 1, \ldots, P$ may be determined via

$$\underline{R}\;\underline{a} = -\underline{r}$$

$P \times P$

Toeplitz-Hermitian with 1^{st} column:

$$\begin{bmatrix} r_{xx}[0] \\ r_{xx}[1] \\ \vdots \\ r_{xx}[P-1] \end{bmatrix}$$

$P \times 1$

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_P \end{bmatrix}$$

$P \times 1$

$$\begin{bmatrix} r_{xx}[1] \\ r_{xx}[2] \\ \vdots \\ r_{xx}[P] \end{bmatrix}$$

$$\sigma_w^2 = r_{xx}[0] + \sum_{k=1}^{p} a_k \underbrace{r_{xx}[-k]}_{r_{xx}^*[k]}$$

$$S_{xx}(\omega) = \frac{\sigma_w^2}{\left|1 + \sum_{k=1}^{p} a_k e^{-jk\omega}\right|^2}$$

- See matlab demos for Session 27