# EE538 DSPI

# Module 23

## Outline:

- Analysis of windowing
  - Sect. 8.2.2, Sect. 12.1.1
- see windowsine.m and windowseg.m
- Estimation of autocorrelation and power spectrum for random signals
  - Sect. 12.1.2

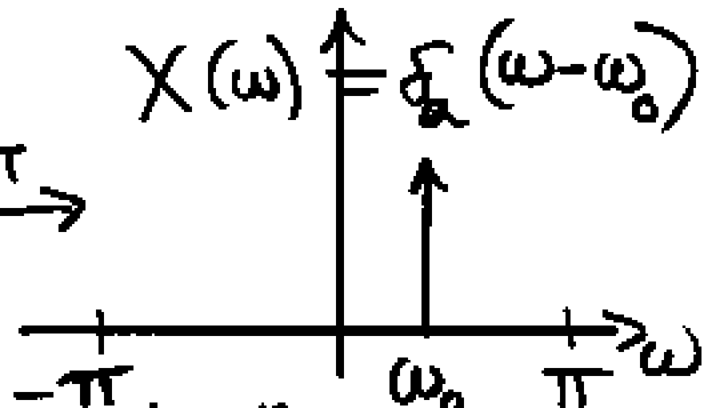
- Analysis of windowing effects
- only have finite length blocks of data in practice
- if data block is extracted from longer stream of data
  ⇒ have effectively multiplied by rectangular window
- to diminish effects of truncation typically employ tapered window in practice

- analysis of effects of windowing with illustrative example of a sum of sinewaves

- Recall:

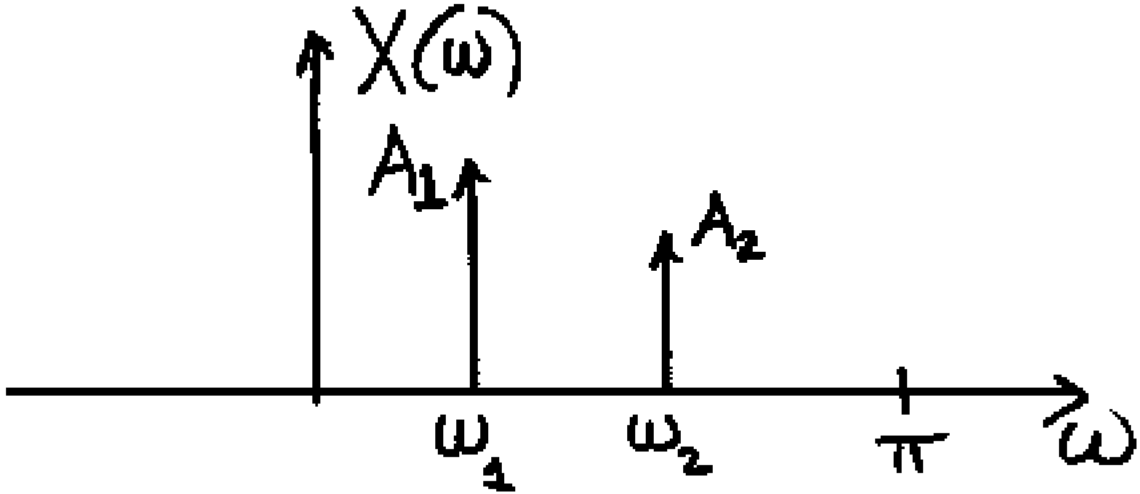
$$x[n] = e^{j\omega_0 n} \quad \overset{\text{DTFT}}{\longleftrightarrow}$$

$-\infty < n < \infty$




- thus, if $x[n] = A_1 e^{j\omega_1 n} + A_2 e^{j\omega_2 n}$

for $-\infty < n < \infty$



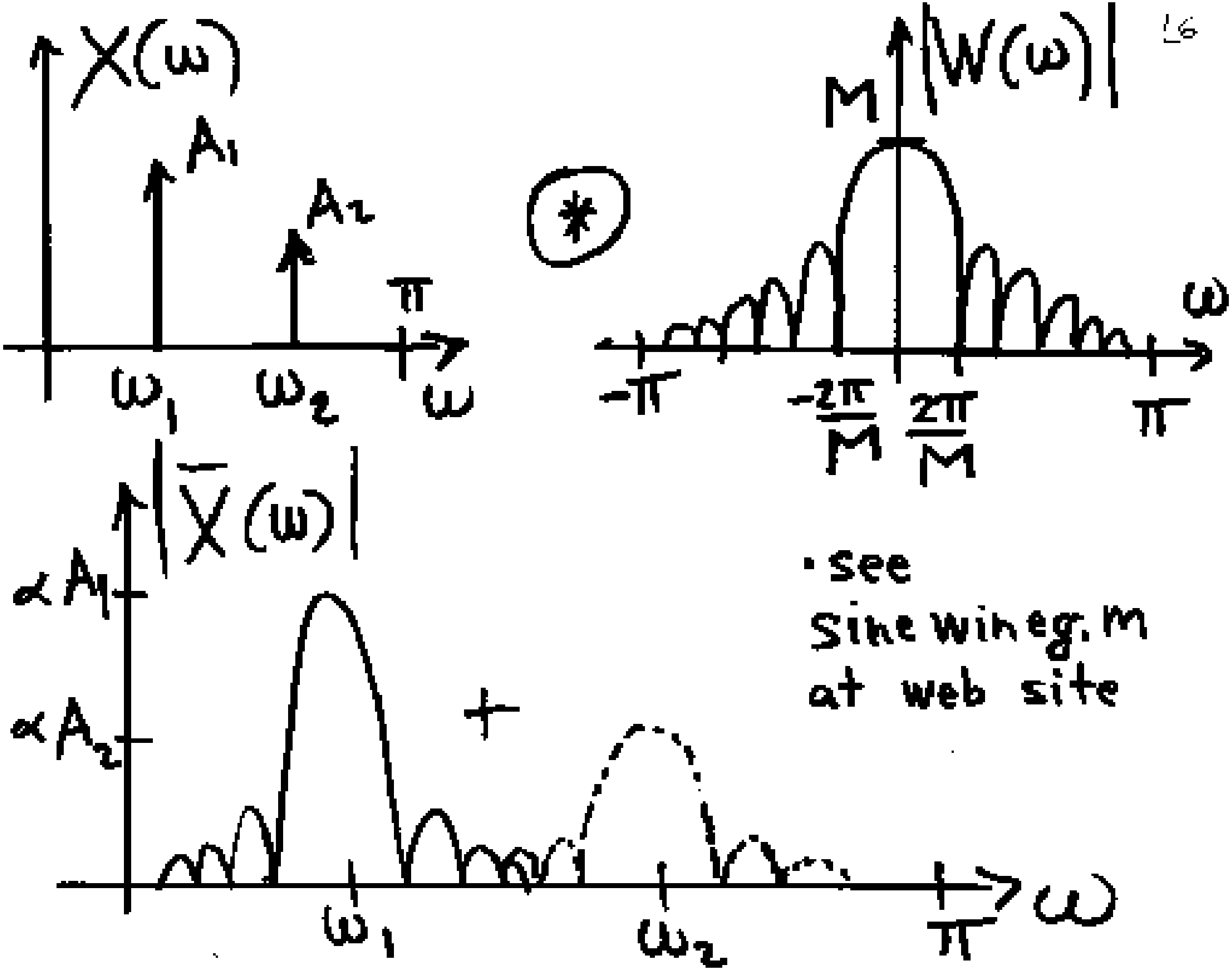

- truncating: $\bar{x}[n] = x[n]\,w[n]$
- where: $w[n] = u[n] - u[n-M]$
  - rectangular window

$$W(\omega) = e^{-j\left(\frac{M-1}{2}\right)\omega} \frac{\sin\left(\frac{M}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$

$$\bar{X}(\omega) = X(\omega) \circledast W(\omega)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\mu) W(\omega - \mu) \, d\mu$$

- periodic convolution




• see sinewineg.m at web site

- mainlobe width of $W(\omega)$ affects resolution of sinewaves closely-spaced in frequency
- sidelobes of $W(\omega)$ caused by stronger sinusoidal component can "mask" presence of weaker sinusoidal components

- Examples of Tapered Windows
- Sine window

DSP book: $w[n] = \sin\left(\frac{\pi}{M-1} n\right)$

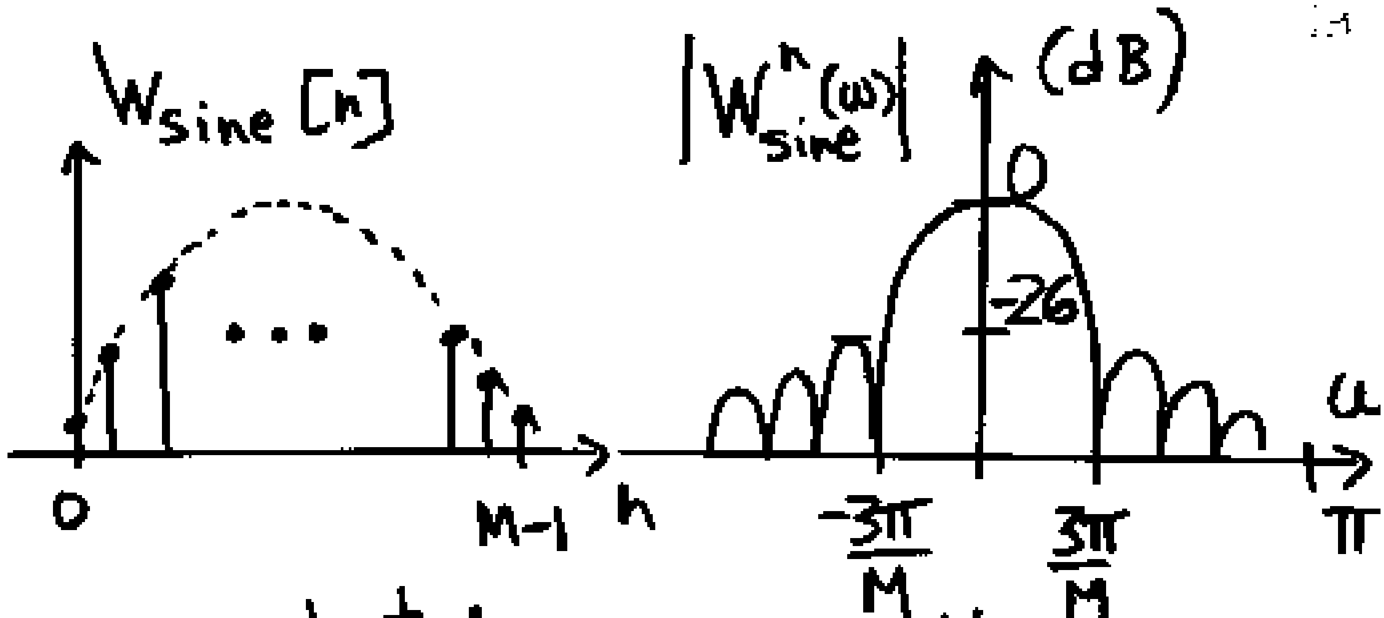
$n = 0, 1, \ldots, M-1$

- alternative form (not equivalent):

$w[n] = \sin\left(\frac{\pi}{M}(n+.5)\right)$

$n = 0, 1, \ldots, M-1$

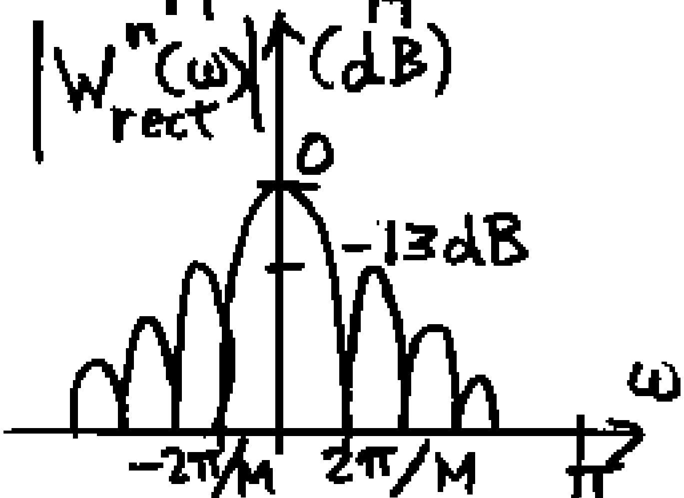
$= \cos\left(\frac{\pi}{M}\left(n - \frac{M-1}{2}\right)\right)$



$w_{sine}[n]$
0
M-1
n
$|W_{sine}^{n}(\omega)|$ (dB)
0
-26
ω
$-\frac{3\pi}{M}$
$\frac{3\pi}{M}$
π


• compared to:

$w_{rect}[n]$
M-1
0
n

$|W_{rect}^{n}(\omega)|$ (dB)
0
-13dB
ω
-2π/M
2π/M
π

- Analyze sidelobe reduction effect
- recall: $e^{j\omega_0 n} w[n] \xleftrightarrow{\text{DTFT}} W(\omega-\omega_0)$
- $w_{sine}[n] = \sin\left(\frac{\pi}{M}\left(n+\frac{1}{2}\right)\right) w_{rect}[n]$

$$= \frac{1}{2j} e^{j\frac{\pi}{2M}} e^{j\frac{\pi}{M}n} w_{rect}[n]$$

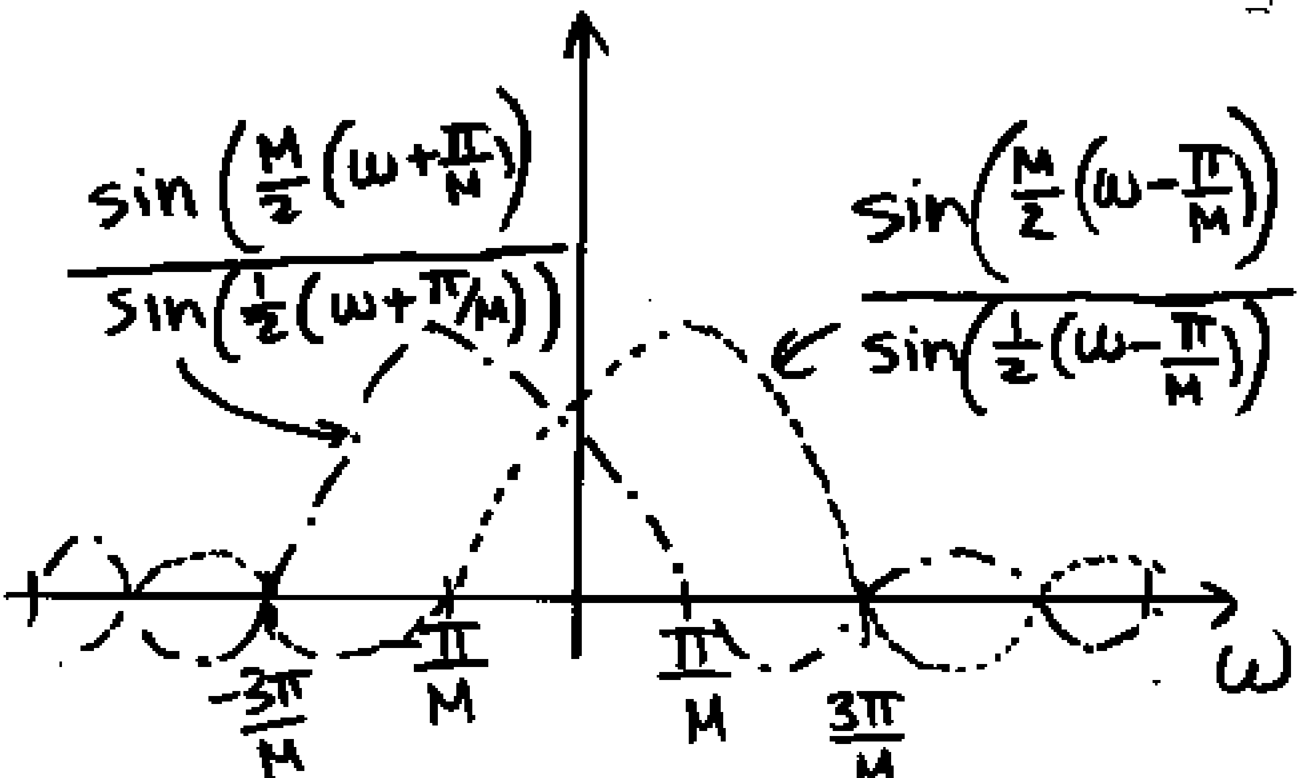
$$- \frac{1}{2j} e^{-j\frac{\pi}{2M}} e^{-j\frac{\pi}{M}n} w_{rect}[n].$$

$$W_{sine}(\omega) =$$

$$\frac{1}{2} e^{j\left(\frac{\pi}{2M} - \frac{\pi}{2}\right)} \frac{\sin\left(\frac{M}{2}\left(\omega - \frac{\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega - \frac{\pi}{M}\right)\right)} e^{-j\left(\frac{M-1}{2}\right)\left(\omega - \frac{\pi}{M}\right)}$$

$$+ \frac{1}{2} e^{j\left(-\frac{\pi}{2M} + \frac{\pi}{2}\right)} \frac{\sin\left(\frac{M}{2}\left(\omega + \frac{\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega + \frac{\pi}{M}\right)\right)} e^{-j\left(\frac{M-1}{2}\right)\left(\omega + \frac{\pi}{M}\right)}$$

$$= \frac{1}{2} e^{-j\left(\frac{M-1}{2}\right)\omega} \left\{ \frac{\sin\left(\frac{M}{2}\left(\omega - \frac{\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega - \frac{\pi}{M}\right)\right)} + \frac{\sin\left(\frac{M}{2}\left(\omega + \frac{\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega + \frac{\pi}{M}\right)\right)} \right\}$$




• sidelobes are 180° out-of-phase
when summed, they cancel

- Hanning & Hamming Windows

$$w[n] = c_0 - c_1 \cos\left(\frac{2\pi}{M}(n+.5)\right)$$

$$0 \le n \le M-1$$

Hanning: $c_0 = c_1 = .5$

Hamming: $c_0 = .54$, $c_1 = .46$

$$w[n] = c_0 \, W_{rect}[n]$$

$$-\frac{c_1}{2} e^{j\frac{\pi}{M}} e^{j\frac{2\pi}{M}} W_{rect}[n] - \frac{c_1}{2} e^{-j\frac{\pi}{M}} e^{-j\frac{2\pi}{M}} W_{rect}[n]$$

- Hanning & Hamming Windows

$$w(n) = c_0 - c_1 \cos\left[\frac{2\pi}{M}(n+.5)\right] \quad 0 \le n \le M-$$

$$= c_0 \, w_{rect}(n) - \frac{c_1}{2} e^{j\frac{\pi}{M}} e^{j\frac{2\pi}{M}n} w_{rect}(n)$$

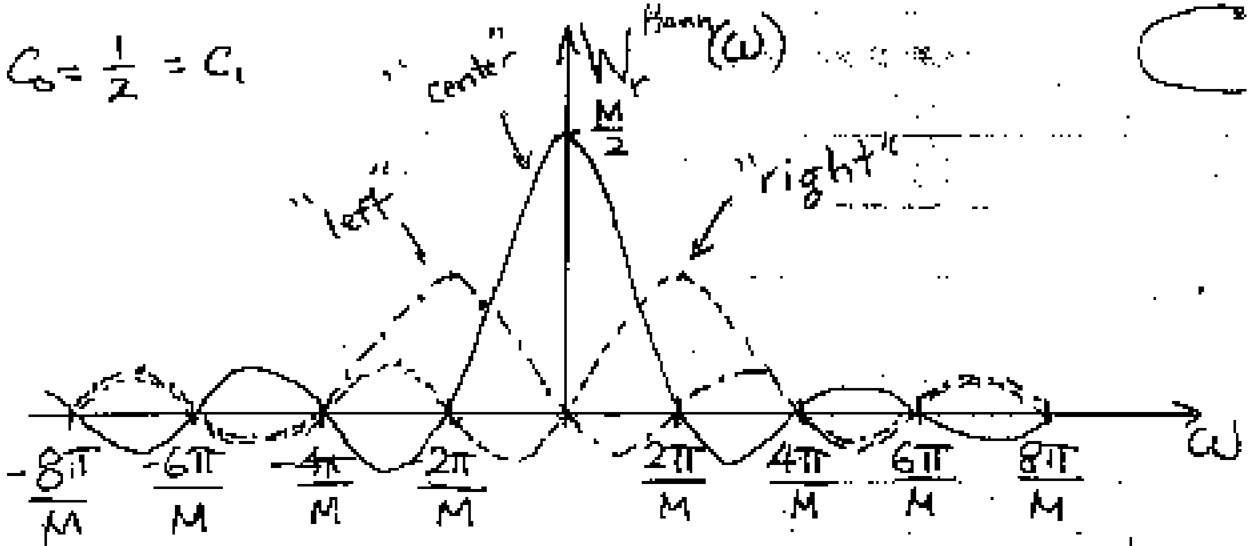
$$- \frac{c_1}{2} e^{-j\frac{\pi}{M}} e^{-j\frac{2\pi}{M}n} w_{rect}(n)$$

note: Hanning : $c_0 = c_1 = .5$

Hamming : $c_0 = .54$, $c_1 = .46$

$$W(\omega) = e^{-j\frac{M-1}{2}\omega} \left\{ \frac{c_1}{2} \frac{\sin\left(\frac{M}{2}\left(\omega + \frac{2\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega + \frac{2\pi}{M}\right)\right)} + c_0 \frac{\sin\left(\frac{M}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} + \frac{c_1}{2} \frac{\sin\left(\frac{M}{2}\left(\omega - \frac{2\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega - \frac{2\pi}{M}\right)\right)} \right\}$$

$C_0 = \frac{1}{2} = C_1$




- sidelobes of "left" and "right" are in-phase
- sum together -- sum patterns sidelobes 180° out-of-phase with "center" pattern
- sum all three patterns to obtain reduced sidelobe levels — mainlobe $8\pi/M$, peak sidelobe $-32$ dB