EE538 DSP I

Module 21

Outline:

- Final comments on "Fast" Computation of DFT: Sect. 8.1.2
- DFT of finite length sinusoid
  – Sect. 7.4
- Inverse DFT – Sect. 7.1.2

- Recall: N-pt. of $x[n]$ is

$$X_N(k) = X(\omega)\Big|_{\omega = \frac{2\pi k}{N},\; k = 0, 1, \ldots, N-1}$$

$$= \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k}{N} n}$$

- if $x[n]$ is of length L, notation assumes $N-L$ zeroes "padded" to $x[n]$

$\Big\}$ Direct computation of N-pt. DFT requires N^2 mults.

- if choose $N = 2^v$ (v, integer) can reduce mults. to $N \log_2(N)$
- However, what if $L = 750$? (length of data block $= L$)
- note: $\underbrace{512}_{2^9} < 750 < \underbrace{1024}_{2^{10}}$
- What about $N = 768 = 3(256)$?
- Any time N is not prime, can reduce no. of mults from N^2 to ?

- Consider $N = ML$, where M and L are integers

$$X_N[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

$$X_N(q+pM) = \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} x[l+mL] \cdot e^{-j\frac{2\pi}{ML}(q+pM)(l+mL)}$$

$$e^{-j\frac{2\pi}{ML}(q+pM)(\ell+mL)}$$

$$= e^{-j\frac{2\pi}{N}q\ell}\, e^{-j\frac{2\pi}{M}mq}\, e^{-j\frac{2\pi}{L}\ell p}\, \underbrace{e^{-j2\pi pm}}_{1}$$

$$X_N[q+pM] = \sum_{\ell=0}^{L-1}\left\{ e^{-j\frac{2\pi}{N}q\ell}\left[\sum_{m=0}^{M-1} x[\ell+mL]\, e^{-j\frac{2\pi}{M}mq}\right]\right\} e^{-j\frac{2\pi}{L}\ell p}$$

- Multiplication Count:

I. L M-pt. DFT's

$\Rightarrow$ requires LM^2 mults.

II. requires $N = ML$ mults.

III. M L-pt. DFT's

$\Rightarrow$ requires $M L^2$ mults.

- Total: $LM^2 + ML + ML^2$

$$= N(M + L + 1) < N^2$$

- Example: $N = 15 \qquad M = 3, \; L = 5$

  $15(3+5+1) = 135 < 15^2 = 225$

- See Fig. 8.1.3 in P&M Text

- End of lecture Treatment of FFT's

- DFT of finite length sinusoid
- $x[n] = e^{j\omega_0 n}$, $n = 0, 1, \ldots, L-1$

  $= 0$ otherwise

  $= e^{j\omega_0 n} \{ u[n] - u[n-L] \}$
- First, compute DTFT of the rectangular window: $w[n] = u[n] - u[n-L]$
- then use modulation property of DTFT:

$$X(\omega) = W(\omega - \omega_0)$$

$$X_N(k) = X(\omega)\Big|_{\omega = \frac{2\pi k}{N}}$$

$$k = 0, 1, \ldots, N-1$$

• I. DTFT of $w[n] = u[n] - u[n-L]$

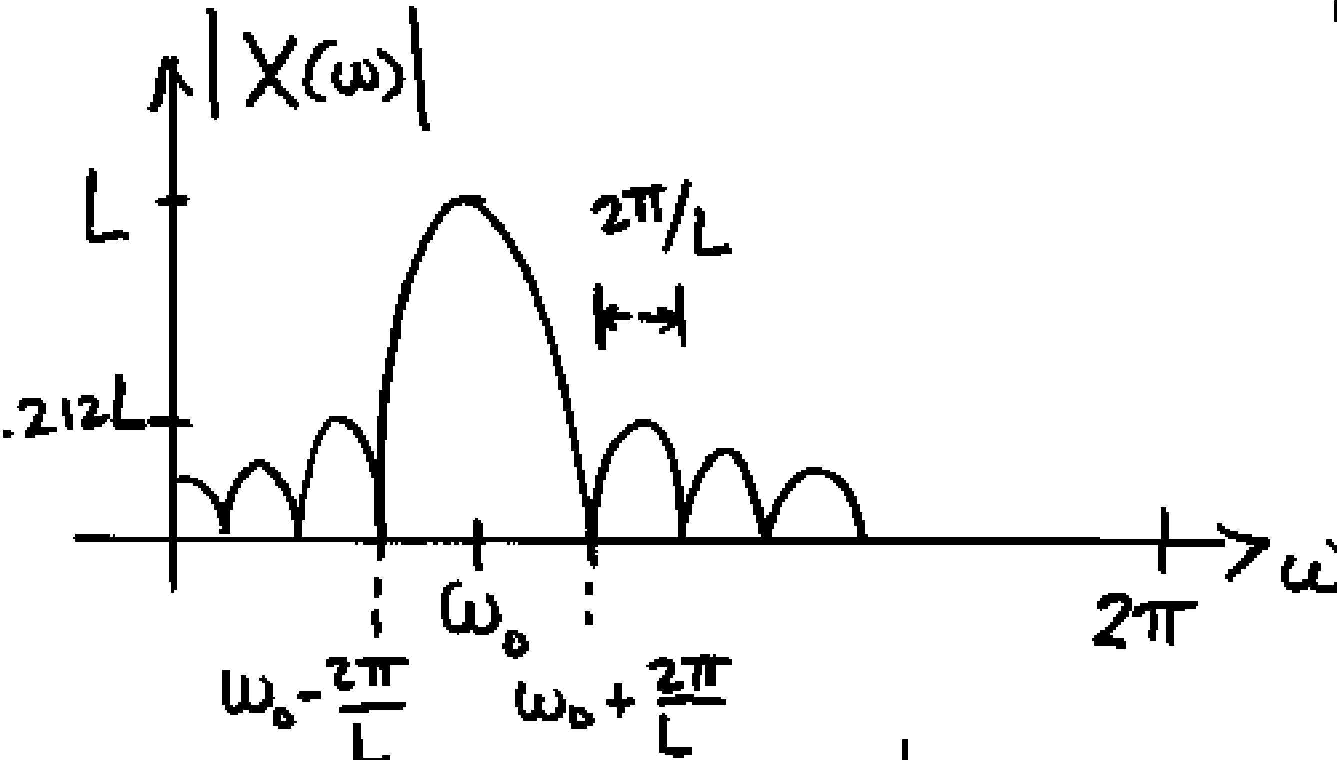
$$W(\omega) = \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{e^{-j\frac{L}{2}\omega}}{e^{-j\frac{\omega}{2}}} \cdot \frac{\left(e^{j\frac{L}{2}\omega} - e^{-j\frac{L}{2}\omega}\right)}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} \cdot \frac{\frac{1}{2j}}{\frac{1}{2j}}$$

$$W(\omega) = e^{-j\frac{(L-1)}{2}\omega} \frac{\sin\left(\frac{L}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$

- thus: DTFT of finite length sinewave:

$$X(\omega) = e^{-j\frac{(L-1)}{2}(\omega-\omega_0)} \frac{\sin\left(\frac{L}{2}(\omega-\omega_0)\right)}{\sin\left(\frac{1}{2}(\omega-\omega_0)\right)}$$



• Finally: $X_N(k) = X(\omega)\Big|_{\omega = \frac{2\pi k}{N}}$

$k = 0, 1, \ldots, N-1$

- Consider three cases:

1.) $\omega_0 = 2\pi \frac{k_0}{L}$ and $N = L$

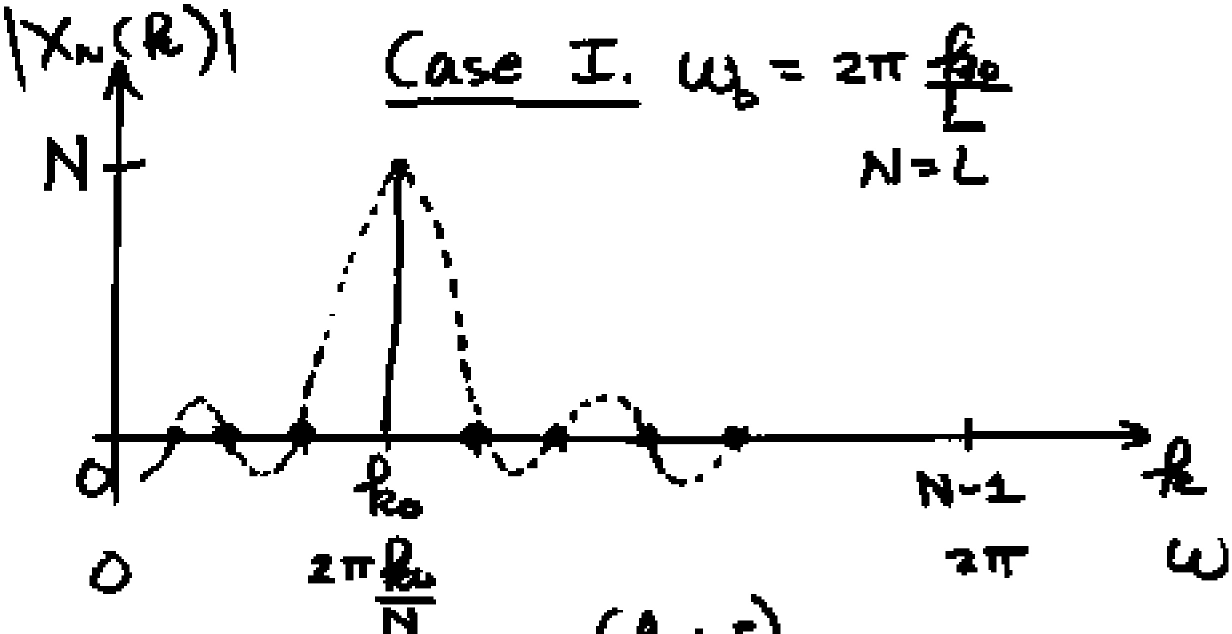
2.) $\omega_0 = 2\pi \frac{k_0}{L}$ and $N = 2L$

3.) $\omega_0 = \frac{2\pi(k_0 + .5)}{L}$ and $N = L$

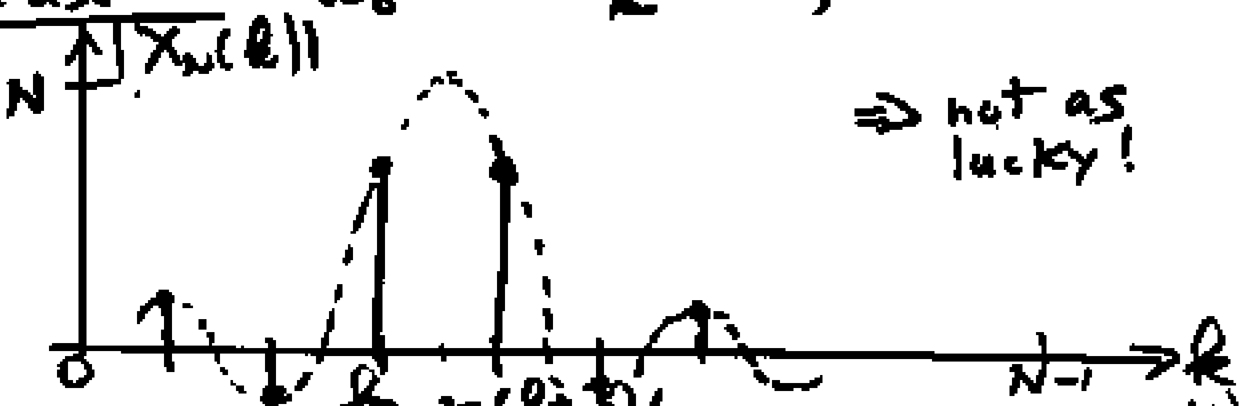
- See sineDFTeg1.m, sineDFTeg2.m, sineDFTeg3.m } at course web site

Case I. $\omega_0 = 2\pi \frac{k_0}{L}$

$N = L$



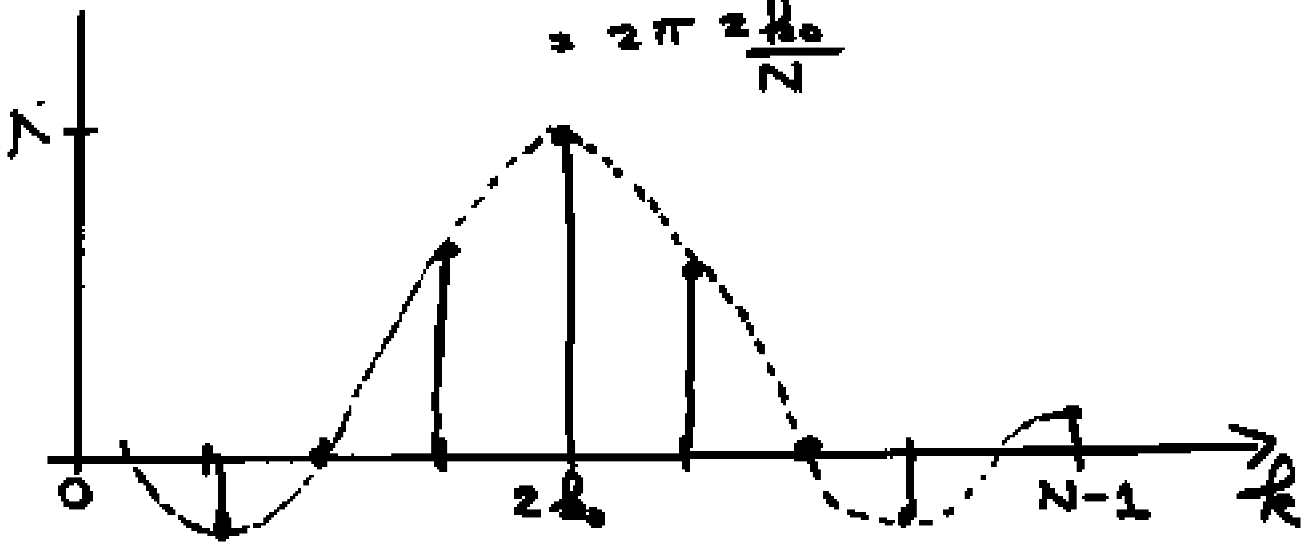
Case 2. $\omega_0 = 2\pi \frac{(k_0 + .5)}{L}$, $N = L$



$\Rightarrow$ not as lucky!

· <u>Case 3.</u> $\omega_0 = 2\pi \frac{k_0}{L}$, $N = 2L$

$= 2\pi \frac{2k_0}{N}$



- as you increase N over L, get a better and better "picture" of DTFT $X(\omega)$ which has both mainlobe and sidelobes ⇒ only get Dirac Delta function with $L \to \infty$ infinite length sinewave