Module 1b

Outline:

- DT Systems: Classifications
  I/O Relations
  Properties
- Relevant Sects. in P&M Text:
  Sect. 2.2 and 2.3
- DT Systems described by difference eqns. => Sect. 2.4

DT System:

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n] = T\left[x[n]\right]$$

- Potential Props. of DT Systems:

- Linearity: $x_1[n] \longrightarrow \boxed{T} \longrightarrow y_1[n]$

$$x_2[n] \longrightarrow \boxed{T} \longrightarrow y_2[n]$$

$$a_1 x_1[n] + a_2 x_2[n] \longrightarrow \boxed{T} \longrightarrow a_1 y_1[n] + a_2 y_2[n]$$

T is linear if this relations holds for any and all a_1 & a_2, and any and all $x_1[n]$ and $x_2[n]$

<u>Time-Invariance</u> (TI)

A system is TI iff

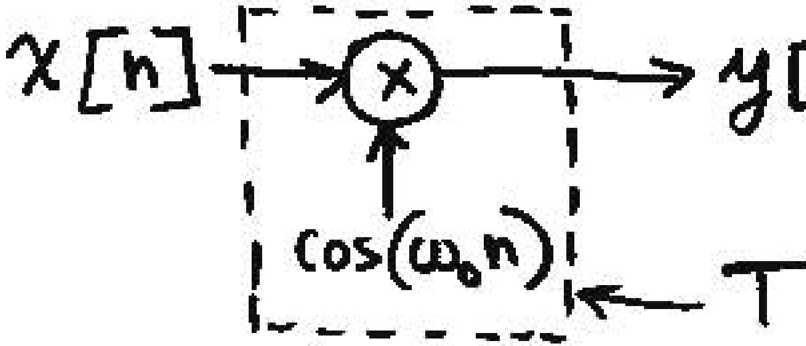
$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n]$$

then

$$x[n-n_0] \longrightarrow \boxed{T} \longrightarrow y[n-n_0]$$

for any and all n_0 and $x[n]$

- **Examples:** 1. Modulation



System is Linear !

$$a_1 x_1[n] + a_2 x_2[n]$$

$$\xrightarrow{T} \left(a_1 x_1[n] + a_2 x_2[n]\right)\cos(\omega_0 n)$$

$$= a_1 x_1[n]\cos(\omega_0 n) + a_2 x_2[n]\cos(\omega_0 n)$$

$$= a_1 y_1[n] + a_2 y_2[n]$$

- System is <u>not</u> TI

$$
\begin{tikzcd}
x[n-n_0] \arrow[r] & \bigotimes \arrow[r] & \cos(\omega_0 n)\, x[n-n_0] \\
 & \cos(\omega_0 n) \arrow[u] &
\end{tikzcd}
$$

$$\cos(\omega_0 n)\, x[n-n_0] \neq y[n-n_0]$$

where: $y[n] = x[n] \cos(\omega_0 n)$

- Example 2.

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n] = x^2[n]$$

- System is not linear
  but is time-invariant

Example 3. Decimation

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n] = x[2n]$$

- effectively halves the original sampling rate
- System is linear but <u>not</u> TI

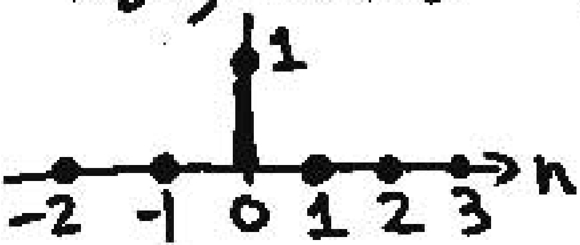
• Example 4. Unit Delay

$$x[n] \longrightarrow \boxed{z^{-1}} \longrightarrow y[n] = x[n-1]$$

System is LTI!

- Define: impulse response of DT System

$$x[n] = \delta[n] \longrightarrow \boxed{T} \longrightarrow y[n] \triangleq h[n]$$



- represent any $x[n]$ as

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$= \cdots + x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + \cdots$$

See Fig. 2.22 on pg. 74

For Fourth Edition, See Fig. 2.3.1 on pg 72

- Derive convolution I/O relation for an LTI System

$$y[n] = T[x[n]]$$

$$= T\left[\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right]$$

$$= \sum_{k} x[k]\,T\left[\delta[n-k]\right] \qquad L$$

$$= \sum_{k} x[k]\,h[n-k] \qquad TI$$

$$= x[n] * h[n] \quad \text{Convolution!}$$

See Fig. 2.23 on pg. 78

and Fig. 2.24 on pg. 81

for graphical convolution egs.

For 4th Edition, see Fig. 2.3.2 on pg 76 and Fig 2.3.3 on pg 79

- Further Properties / Classifications of LTI System:
- Finite Impulse Response (FIR)

$$h[n] = \begin{cases} 0, & n \le n_1 \\ 0, & n \ge n_2 \end{cases} \quad -\infty < n_1 < n_2 < \infty$$

- Infinite Impulse Response (IIR)
  - if either $n_1 = -\infty$ or $n_2 = \infty$

- **Stability**: A DT System is bounded input-bounded output (BIBO) stable

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n]$$

If $|x[n]| \le M_i$ for all n $(M_i < \infty)$, then system is BIBO Stable when $|y[n]| \le M_o$ for all n $(M_o < \infty)$

Theorem: LTI System is BIBO stable iff

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

See pp. 88-89 for proof

4th Edition, see pg 86

Causality: present output does not depend on future inputs

Recall for LTI : $y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

- since convolution is commutative
- $$y[n] = x[n] * h[n]$$
$$= h[n] * x[n]$$
$$= \sum_{k} h[k]\, x[n-k]$$
$$= \dots h[-2]\, x[n+2] + h[-1]\, x[n+1]$$
$$+ h[0]\, x[n] + h[1]\, x[n-1] + \dots$$
- LTI system is causal iff $h[n] = 0$ for $n < 0$

- LTI Systems Described by Difference Equations

$$y[n] = -\sum_{k=1}^{N} a_k\, y[n-k] + \sum_{k=0}^{M} b_k\, x[n-k]$$

- if $N=0 \Rightarrow$ FIR
- otherwise $\Rightarrow$ IIR

See notcheg.m demo at web site