EE538 DSPI

Module 16

Outline:

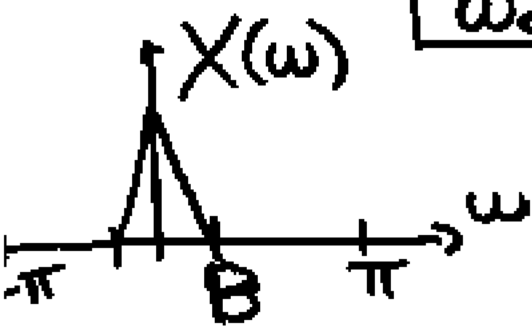
- Final comments on IIR Filter Design - scanned in at web site
- FIR vs. IIR Digital Filter Design - Sect. 8.6
- Design of Linear-Phase FIR Filters - Sect. 8.2.4

Digital Filter Design

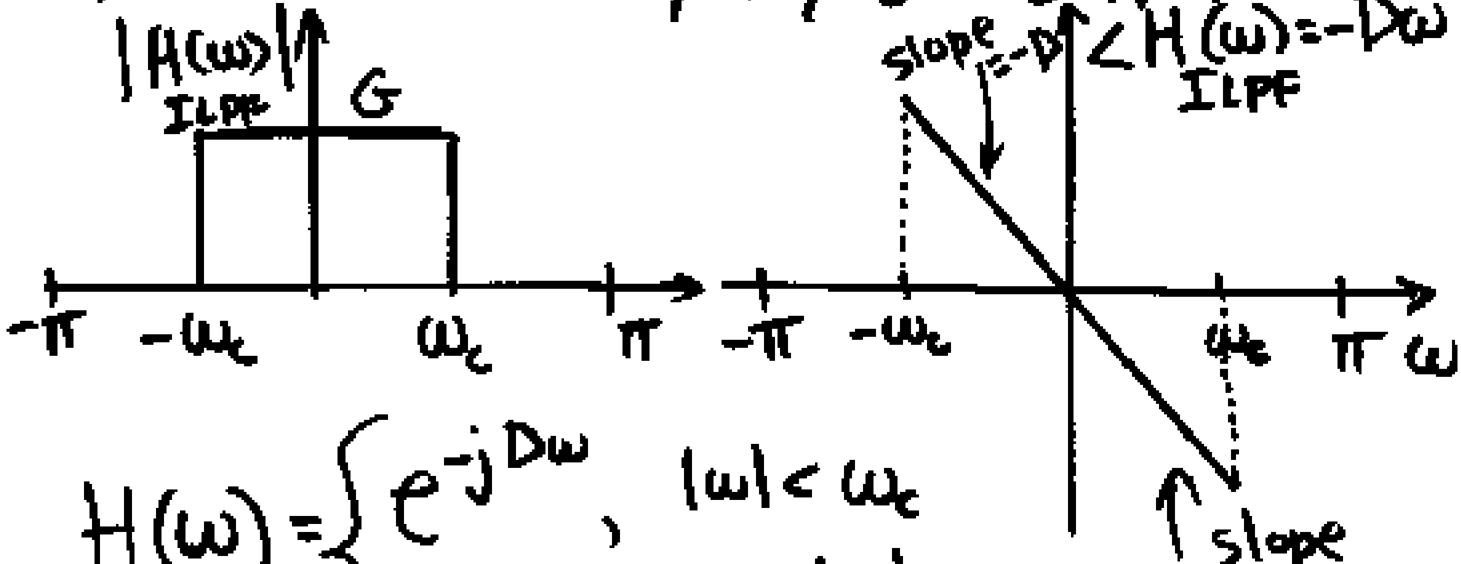
- Frequency Selective Linear Filtering $\Rightarrow$ Sect. 4.5
- Ideal Lowpass Filter Characteristics

$$x(n) \longrightarrow \boxed{\begin{matrix}\text{ILPF} \\ \omega_c > B\end{matrix}} \longrightarrow y(n) = G\,x(n-D)$$

D, integer



from Shift Property of DTFT



$$H_{ILPF}(\omega) = \begin{cases} e^{-jD\omega}, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

- What if slope of phase is not an integer? Not a problem!

$$x_a(t) \longrightarrow \boxed{\begin{matrix}\cancel{\text{DAC}}\ \text{A/D} \\ F_s\end{matrix}} \xrightarrow{x(n)} \boxed{\begin{matrix}\text{ILPF} \\ \omega_c\end{matrix}} \xrightarrow{y(n)} \boxed{\begin{matrix}\text{DAC} \\ F_s\end{matrix}} \longrightarrow x_a(t - DT_s)$$

$F_s > 2B$

$\angle H(\omega) = -D\omega$

D not integer

$$2\pi \frac{B}{F_s} \le \omega_c < \pi$$

$$T_s = \frac{1}{F_s}$$

$$y(n) = x_a(t - DT_s)\Big|_{t = nT_s = \frac{n}{F_s}}$$

but $y(n) \neq x(n-D)$

if D is not an integer

Impulse Response of ILPF

$$h_{ILPF}(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega D} e^{j\omega n}$$

$$h_{ILPF}(n) = \frac{\sin[\omega_c(n-D)]}{\pi(n-D)}$$

$$-\infty < n < \infty$$

- Practical Problems:
  - non- causal
  - infinite length

- approximate ILPF with
- IIR filter via difference eqn.

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

OR

- FIR filter via :

$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k)$$

$$= \sum_{k=0}^{M} h(k)\, x(n-k)$$

$h(n) = b_n$

$n = 0, 1, \ldots, M$

- <u>note:</u> IIR Filters generally have nonlinear phase
- trick for achieving linear phase with IIR filters off-line in non real-time applications
- $$x[n] \longrightarrow \boxed{\begin{matrix}\text{IIR}\\ h[n]\end{matrix}} \longrightarrow z[n]$$

$$z[-n] \longrightarrow \boxed{\begin{matrix}\text{IIR}\\ h[n]\end{matrix}} \longrightarrow w[n]$$

$$y[n] = w[-n]$$

- analysis: $Z(\omega) = H(\omega) X(\omega)$

$$W(\omega) = H(\omega) \{ Z^*(\omega) \}$$

- since: $z[-n] \xleftrightarrow{DTFT} Z^*(\omega)$

$$W(\omega) = H(\omega) H^*(\omega) X^*(\omega)$$
$$= |H(\omega)|^2 X^*(\omega)$$

- since $y[n] = w[-n]$

$$Y(\omega) = W^*(\omega) = |H(\omega)|^2 X(\omega)$$

- Design of Linear-Phase FIR Filters
- constrain $h[n]$ to be symmetric
- e.g., consider M to be odd

$$y[n] = \sum_{k=0}^{M-1} h[k]\, x[n-k]$$

- symmetry constraint:

$$h[M-1-n] = h[n], \quad n = 0, 1, \ldots, M-1$$

- show symmetry guarantees linear phase

$$H(\omega) = h[0] + h[1]e^{-j\omega} + \ldots + h\left[\tfrac{M-1}{2}\right]e^{-j\frac{M-1}{2}\omega} + \ldots + h[M-2]e^{-j(M-2)\omega} + h[M-1]e^{-j(M-1)\omega}$$

- <u>note:</u> $e^{-jn\omega} + e^{-j(M-1-n)\omega}$

$$= e^{-j\left(\frac{M-1}{2}\right)\omega} \left\{ e^{+j\left(\frac{M-1}{2}-n\right)\omega} + e^{-j\left(\frac{M-1}{2}-n\right)\omega} \right\}$$

$$= e^{-j\frac{(M-1)}{2}\omega} \; 2\cos\left[\left(\frac{M-1}{2}-n\right)\omega\right]$$

- ultimately:

$$H(\omega) = H_r(\omega)\, e^{-j\frac{M-1}{2}\omega}$$

$$H_r(\omega) = h\left[\frac{M-1}{2}\right] + 2\sum_{n=0}^{(M-3)/2} h[n]\cos\left[\left(\frac{M-1}{2} - n\right)\omega\right]$$

- note: $H_r(\omega)$ is real-valued but possibly negative

$$\angle H(\omega) = \begin{cases} -\left(\frac{M-1}{2}\right)\omega, & \text{if } H_r(\omega) > 0 \\ \pi - \left(\frac{M-1}{2}\right)\omega, & \text{if } H_r(\omega) < 0 \end{cases}$$

• change of variables:

$$k = \frac{M-1}{2} - n \implies n = \frac{M-1}{2} - k$$

• new limits: $$k \Big|_{\frac{M-1}{2}}^{\frac{M-1}{2} - \frac{M-3}{2} = 1}$$

$$H_r(\omega) = h\left(\frac{M-1}{2}\right)\cos(0\cdot\omega) + \sum_{k=1}^{(M-1)/2} 2h\left[\frac{M-1}{2} - k\right]\cos(k\omega)$$

$$H_r(\omega) = \sum_{k=0}^{(M-1)/2} \alpha[k] \cos(k\omega)$$

- where:

$$\alpha[k] = \begin{cases} h\left[\frac{M-1}{2}\right], & k=0 \\ 2h\left[\frac{M-1}{2}-k\right], & k=1, \dots, \frac{M-1}{2} \end{cases}$$

- For M even, see Text Table 8.5 on pg. 641