EE538 DSPI

Module 15

Outline:

- Bilinear Transform Examples from old Exam 3's

- Design of IIR Filters via Bilinear Transform
- Example 1.
- 2^{nd} order Butterworth LPF with a 3-dB cut-off at $\Omega_c = 1$ has the transfer function

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$$

Use bilinear transform

$$s = c \, \frac{z-1}{z+1}$$

to design a digital filter with 3 dB cut-off at $\omega_c = \pi/2$ rads/sec.

(a) $\Omega_c = c \tan(\omega_c/2)$

$1 = c \tan\left(\frac{\pi}{2}/2\right)$

$\cdot$ since $\tan(\pi/4) = 1 \Rightarrow c = 1$

$$H(z) = H_a(s)\Big|_{s=\frac{z-1}{z+1}}$$

$$= \frac{1}{\frac{(z-1)^2}{(z+1)^2} + \sqrt{2}\,\frac{z-1}{z+1} + 1} \cdot \frac{(z+1)^2}{(z+1)^2}$$

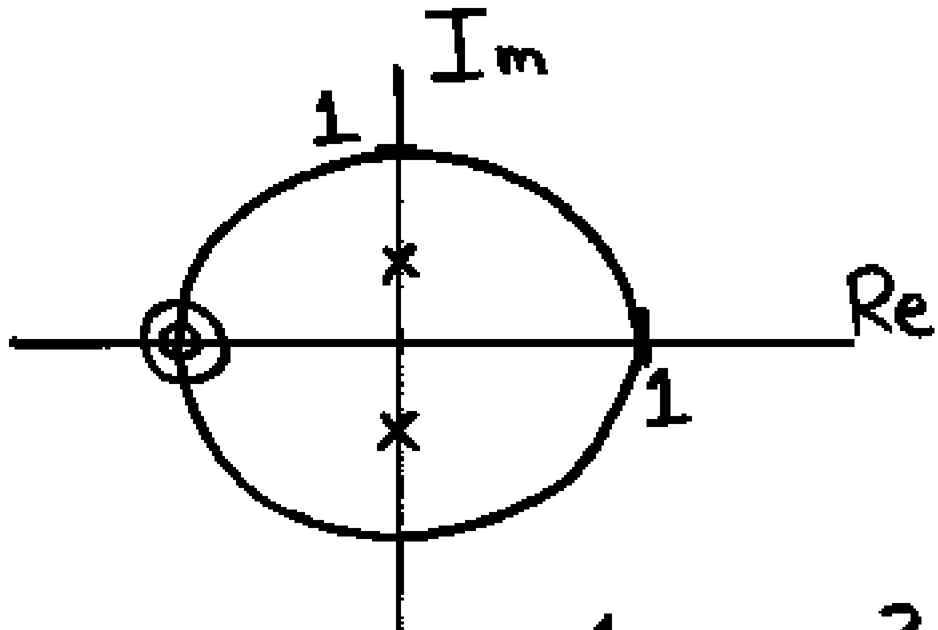
$$= \frac{(z+1)^2}{(z-1)^2 + \sqrt{2}\,(z^2-1) + (z+1)^2}$$

$$H(z) = \frac{(z+1)^2}{z^2 - 2z + 1 + \sqrt{2}\, z^2 - \sqrt{2} + z^2 + 2z + 1}$$

$$= \frac{(z+1)^2}{(2+\sqrt{2})\, z^2 + 2 - \sqrt{2}}$$

• poles: $\quad P_k = \pm j \sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}}$

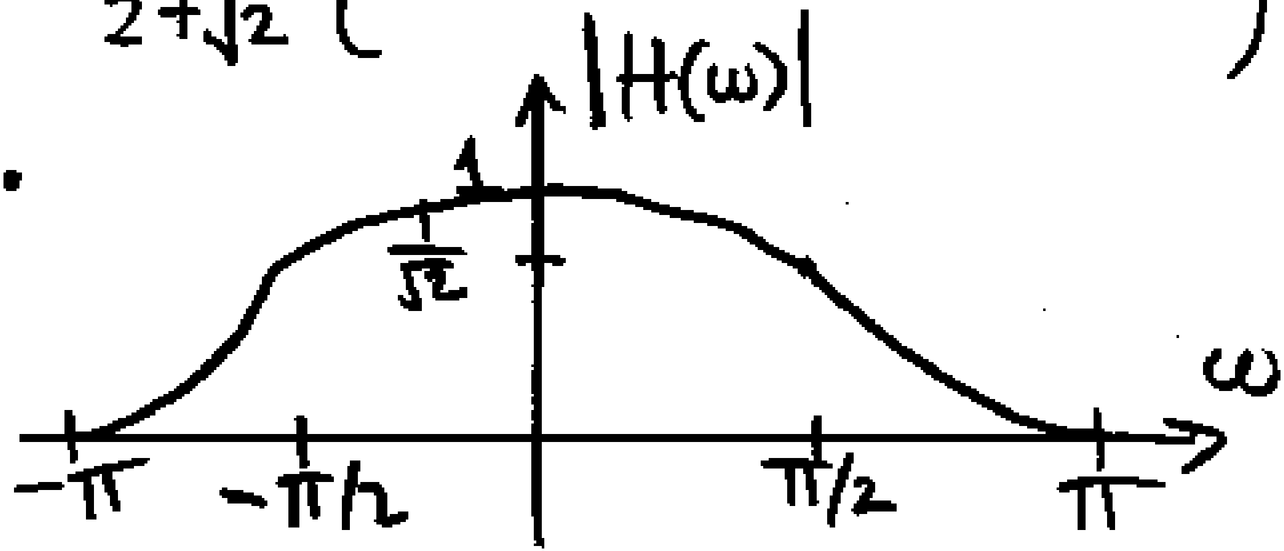
$$= \pm j\, .414$$



$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{2+\sqrt{2} + \left(2-\sqrt{2}\right) z^{-2}}$$

$$= Y(z) / X(z)$$

$$y[n] = -\frac{(2-\sqrt{2})}{2+\sqrt{2}}\, y[n-2] + \frac{1}{2+\sqrt{2}}\left\{x[n] + 2x[n-1] + x[n-2]\right\}$$



Preamble to Demo on Using Bilinear Transform to Design Digital IIR Filters

- matlab command:

ellip.m ; ellipeg.m

Inputs: Filter order (no. of poles)

$\omega_p, \delta_1, \omega_s, \delta_2$

Outputs: coefficients of diff. eqn. for digital filter

- ellipord.m $\Rightarrow$ to determine filter order a-priori

- to illustrate bilinear transform design, demo uses 's' option in ellip.m to design analog elliptic filter from pre-warped passband and stopband edges

$$[\beta, \alpha] = \text{ellip}(\Omega_p, \delta_1, \Omega_s, \delta_2, N, \text{'s'})$$

yields

$$H_a(s) = \frac{\sum_{k=0}^{M_a} \beta_k s^k}{\sum_{k=0}^{N} \alpha_k s^k} \qquad \boxed{\alpha_0 = 1}$$

to illustrate bilinear transform:

$$H(\omega) = H_a(s)\Big|_{s = c\frac{z-1}{z+1}}\Big|_{z = e^{j\omega}}$$

$$H(\omega) = \frac{\displaystyle\sum_{k=0}^{M_a} \beta_k \left(c \, \frac{e^{j\omega}-1}{e^{j\omega}+1} \right)^k}{\displaystyle\sum_{k=0}^{N} \alpha_k \left(c \, \frac{e^{j\omega}-1}{e^{j\omega}+1} \right)^k}$$

- demo uses $c = 1$
- actual analog specs:
  F_p, δ_1, F_{sp}, δ_2, and F_s sampling rate

- digital specs:

$$\omega_p = 2\pi \frac{F_p}{F_s}, \delta_1, \omega_s = 2\pi \frac{F_{sp}}{F_s}, \delta_2$$

- bilinear transform requires frequency pre-warping

$$\Omega_p = \tan\left(\frac{\omega_p}{2}\right) \quad \left(\neq 2\pi F_p\right)$$

$$\Omega_s = \tan\left(\frac{\omega_s}{2}\right) \quad \left(\neq 2\pi F_{sp}\right)$$

- again: ellip.m can output digital filter design directly
  $\Rightarrow$ don't use 's' option

- on
$$[A, B] = \text{ellip}(N, \omega_p, \delta_1, \omega_s, \delta_2)$$

contain filter coefficients

- can use freqz.m to plot freq. response of digital filter given the coeffs. of diff. eqn.
- Prob. 8.16

$F_p = 4$ kHz $F_{sp} = 6$ kHz $F_s = 24$ kHz

$\delta_1 = 1$ dB $\delta_2 > 40$ dB