EE538 DSPI

Module 13

Outline:

- Analysis of Quantization Errors - SQNR Sect. 6.3.3
  - matlab demo - quantizeb2.m

Oversampling D/A Converters (Sigma/Delta Modulation) - Sect 6.6 (6.6.2)

- Analysis of Quantization Errors - see Fig. 9.8 on pg. 751
- dominating source of "noise" in DSP Systems is quantization error
- error control coding can correct errors made in reading the bits off the disk

- quantization error:

$$e[n] = x_a(nT_s) - x_Q(nT_s)$$
$$= x[n] - \underbrace{x_Q[n]}_{\text{quantized sample value}}$$

- Signal to Quantization Noise Ratio (SQNR)

$$SQNR = 10\log_{10}\left\{\frac{E\{x^2[n]\}}{E\{e^2[n]\}}\right\}$$

- $E\{\cdot\}$ denotes expected value in a statistical sense
- Assumptions:
- $e[n]$ is a uniformly distributed random variable over $-\frac{\Delta}{2}$ to $+\frac{\Delta}{2}$
- $E\{e^2[n]\} = \dfrac{\Delta^2}{12}$

B = # of bits per sample

2^B = no. of levels

$(2^B - 1 \approx 2^B)$

- note: if A is the upper limit of the quantizer: $\Delta = 2A/2^B$

$$E\{e^2[n]\} = \frac{1}{12}\left(\frac{2A}{2^B}\right)^2 = \frac{1}{3}\frac{A^2}{2^{2B}}$$

- for signal model, consider two cases:

I. $x[n] = x_a(nT_s)$ is uniformly distributed over $-A$ to $+A$

$$E\{x^2[n]\} = \frac{(2A)^2}{12} = \frac{A^2}{3}$$

$$SQNR = 10 \log_{10} \left\{ \frac{A^2/3}{A^2/3\cdot 2^{2B}} \right\}$$

$$\log\{x^a\} = a \log\{x\}$$

$$SQNR = 2B \log_{10}\{2\}$$

$$= 6.02\,B \quad dB$$

$\Rightarrow$ 6 dB per bit

II. Assume $x[n] = x_a(nT_s)$ is Gaussian distributed over the range of the quantizer with zero mean and

$E\{x^2[n]\} = \sigma_x^2$

- design so that 99% of the time $x_a(nT_s)$ is in the range of the quantizer from $-A$ to $+A$

$$\Rightarrow 6\sigma_x = 2A$$

$$A = 3\sigma_x$$

$$\sigma_x^2 = A^2/9$$

- $SQNR = 10 \log_{10} \left\{ \dfrac{A^2/9}{A^2/3 \cdot 2^{2B}} \right\}$

$$= 10 \log_{10} \left\{ 2^{2B}/3 \right\}$$

$$= 6.02\,B - 4.77 \quad dB$$

- classical result: 6 dB gain in SQNR for each add'l bit allocated to each sample

- CD Players: 16 bits/sample
- Yields approximately 95 dB SQNR
- see demo quantizeb2.m

Single bit DAC:

- in CD players, digitally up sampling rate by L ($=8$, e.g.) just prior to reconstruction
- sampling rate of original recording $F_s = 44.1$ KHz $\Rightarrow 16 \frac{\text{bits}}{\text{sample}}$
- new effective rate $8(44.1 \text{ KHz}) = 352.8$ KHz

- however, rather than store 16 bits per sample at the 352.8 KHz rate (8x as many bits)
- assume at that high a rate, difference between 2 successive samples is either

  $+\Delta$ => encode as 1

  $-\Delta$ => encode as 0

- where Δ is quantizer step size

- Single bit DAC:
  Capacitor voltage is either increased by Δ or decreased by Δ and held from one sample to the next
  - See Fig. 6.6.2 and Fig. 6.6.7
    Fig. 1.4.8 on pg 34

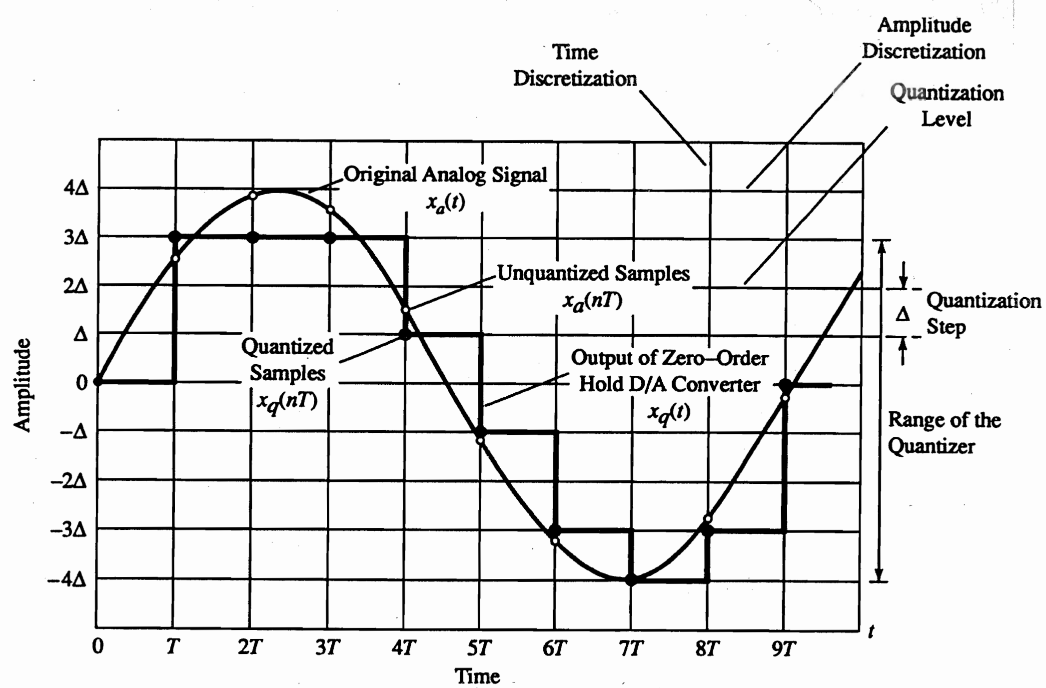


Figure 1.4.8 Sampling and quantization of a sinusoidal signal.

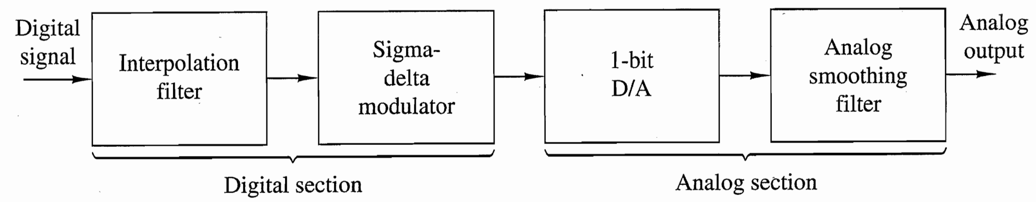


Figure 6.6.7 Elements of an oversampling D/A converter.

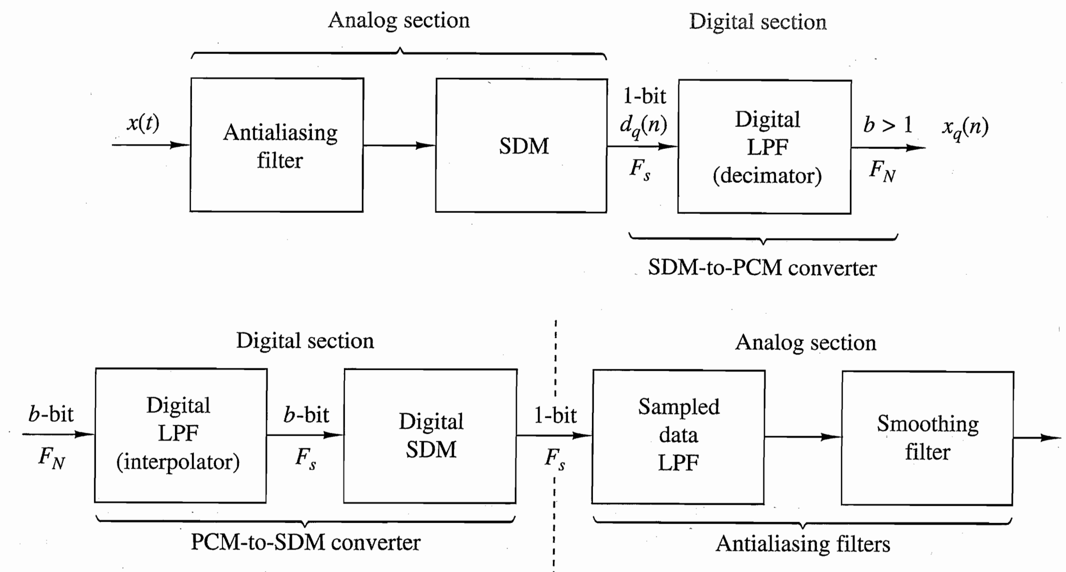
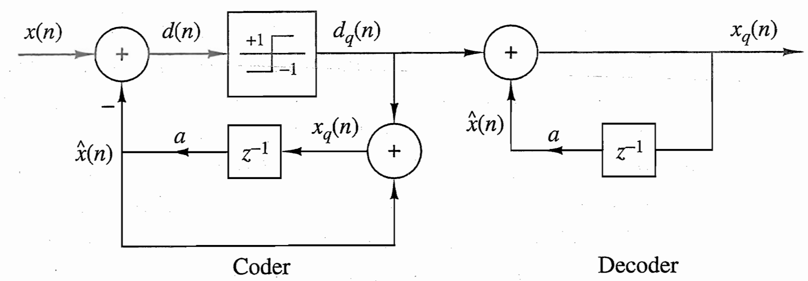
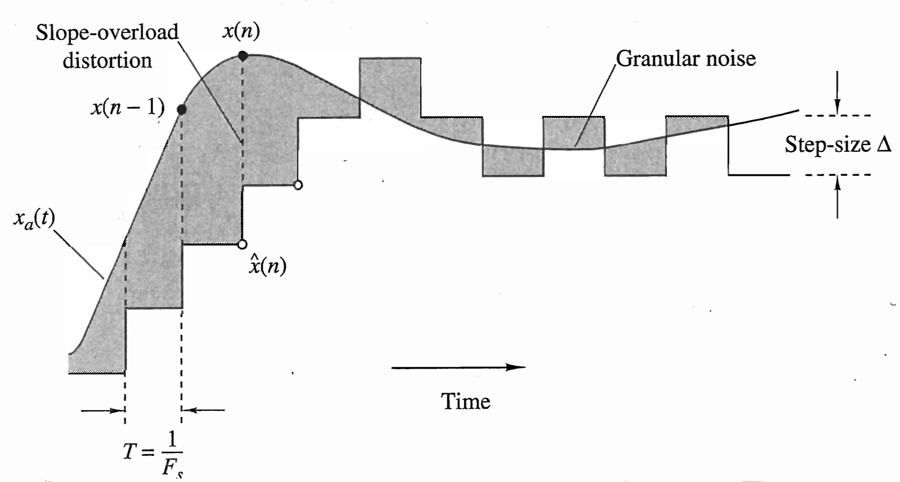


Figure 6.6.6 Basic elements of an oversampling A/D converter.


x(n)
d(n)
+1
−1
$d_q(n)$
$x_q(n)$
−
$\hat{x}(n)$
a
z^{-1}
$x_q(n)$
$\hat{x}(n)$
a
z^{-1}
Coder
Decoder

(a)


Slope-overload
distortion
x(n)
Granular noise
x(n − 1)
Step-size Δ
$x_a(t)$
$\hat{x}(n)$
Time
$T = \frac{1}{F_s}$