EE538 DSP I

Module 11

Outline:

- Efficient downsampling
  Sect. 11.5.2
- Fractional Sampling Rate Conversion
  Sect. 11.4
- Digital Subbanding
  Sect. 11.7.2, 11.9.7

- Math preamble:

$$\sum_{k=-\infty}^{\infty} \delta[n-kL] = \sum_{k=0}^{L-1} \frac{1}{L} e^{j\frac{2\pi kn}{L}}$$

$$= \frac{1}{L} \frac{1 - e^{j2\pi n}}{1 - e^{j2\pi n/L}}$$

$$= \begin{cases} 1, & n = \ell L,\ \ell \text{ integer} \\ 0, & \text{otherwise} \end{cases}$$

- Downsampling:

$$y[n] = x[Dn]$$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} x[Dn] e^{-j\omega n}$$

$$= \sum_{n'=-\infty}^{\infty} x[n'] e^{-j\frac{\omega n'}{D}}$$

$$\boxed{\begin{aligned} n' &= nD \\ n &= \frac{n'}{D} \end{aligned}}$$

$n' = \ell D$

$$= \sum_{n=-\infty}^{\infty} x[n] \frac{1}{D} \sum_{k=0}^{D-1} e^{j\frac{2\pi k n}{D}} e^{-j\frac{\omega n}{D}}$$

$$Y(\omega) = \sum_{k=0}^{D-1} \frac{1}{D} \sum_{n=-\infty}^{\infty} x[n] e^{-j\left(\frac{\omega}{D} - \frac{2\pi k}{D}\right) n}$$

$$Y(\omega) = \frac{1}{D} \sum_{k=0}^{D-1} X\left(\frac{\omega - 2\pi k}{D}\right)$$

• note: if $F_{s_{new}} = \frac{F_{s_{old}}}{D} > 2W$,

then $Y(\omega) = \frac{1}{D} X\left(\frac{\omega}{D}\right)$ for $|\omega| < \pi$

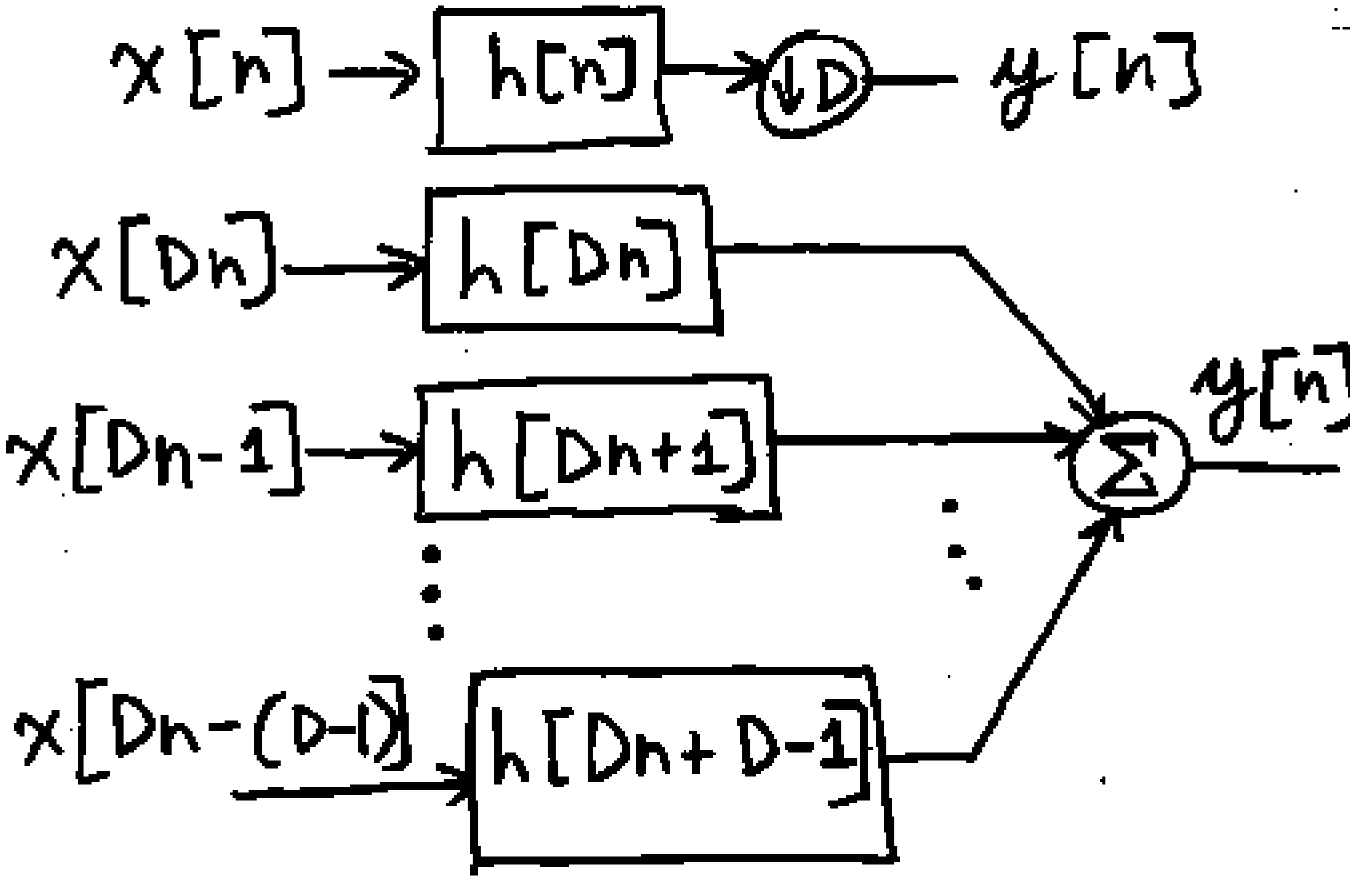
- Efficient Down-Sampling:

$$x[n] \longrightarrow \boxed{h[n]} \xrightarrow{w[n]} \boxed{D\downarrow} \longrightarrow y[n]$$

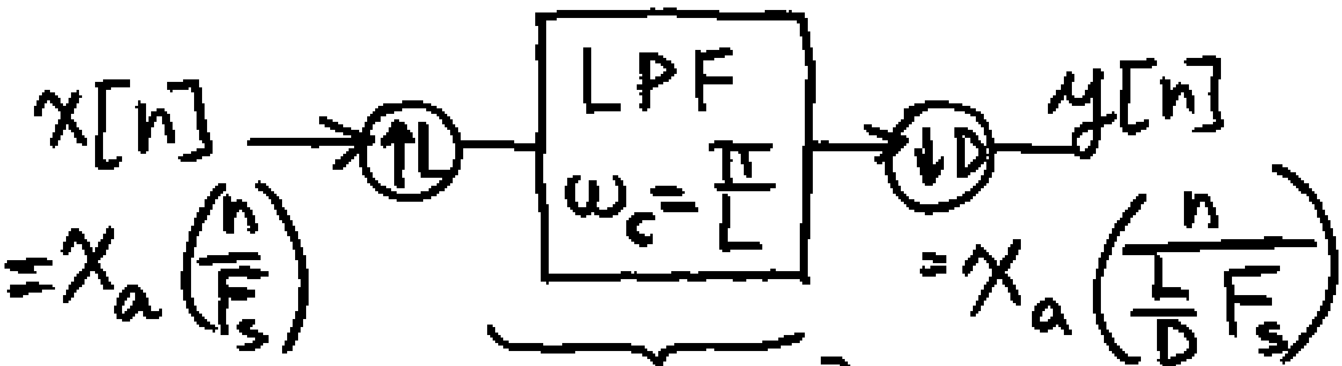
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[Dn-k]$$

$$= \sum_{l=0}^{D-1} \sum_{k'=-\infty}^{\infty} h[k'D+l]\, x[Dn-(k'D+l)]$$

$$= \sum_{l=0}^{D-1} \sum_{k=-\infty}^{\infty} h[kD+l]\, x[D(n-k)-l]$$

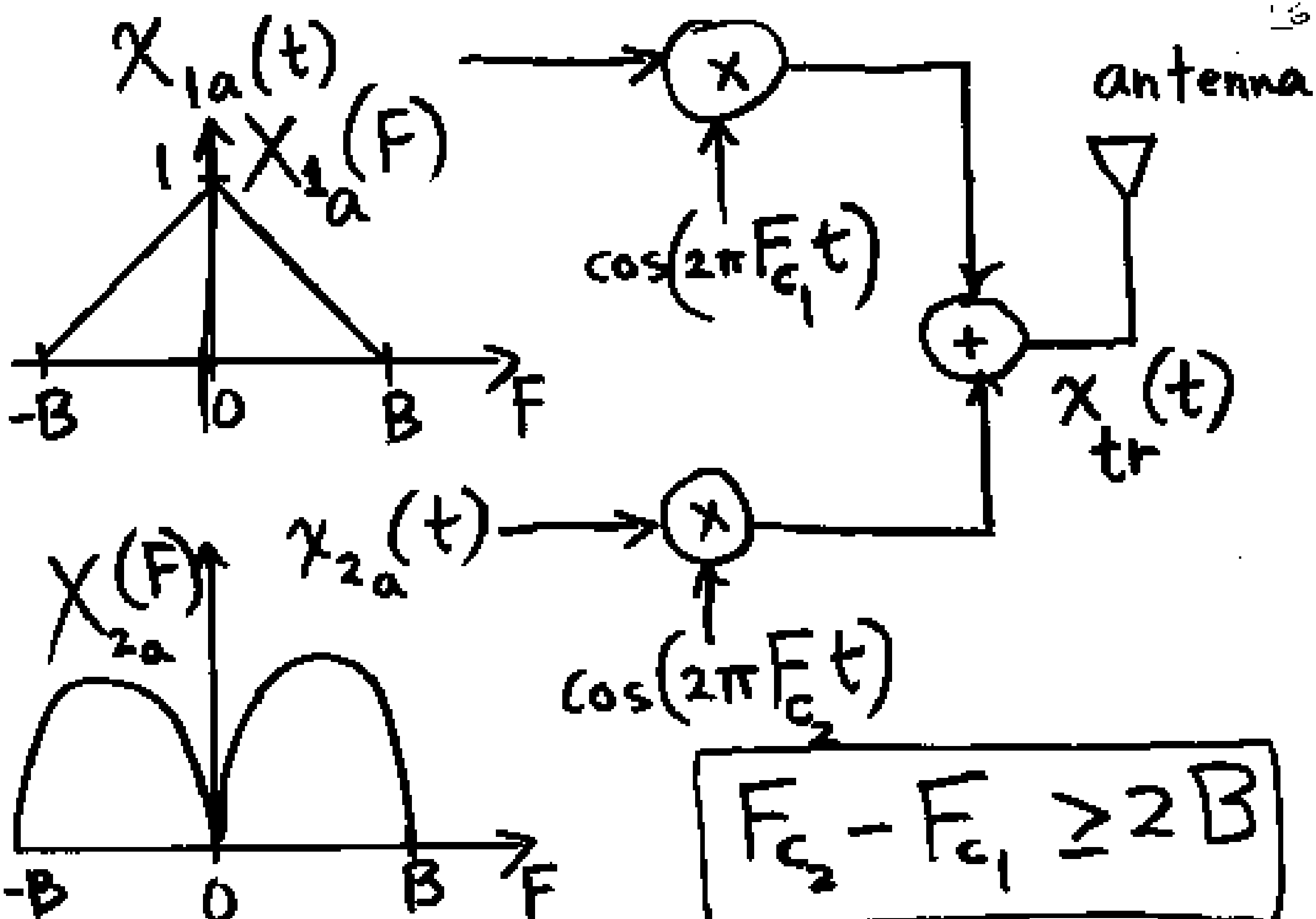

x[n]
h[n]
↓D
y[n]
x[Dn]
h[Dn]
x[Dn-1]
h[Dn+1]
Σ
y[n]
x[Dn-(D-1)]
h[Dn+D-1]

- Fractional Sampling Rate Conversion

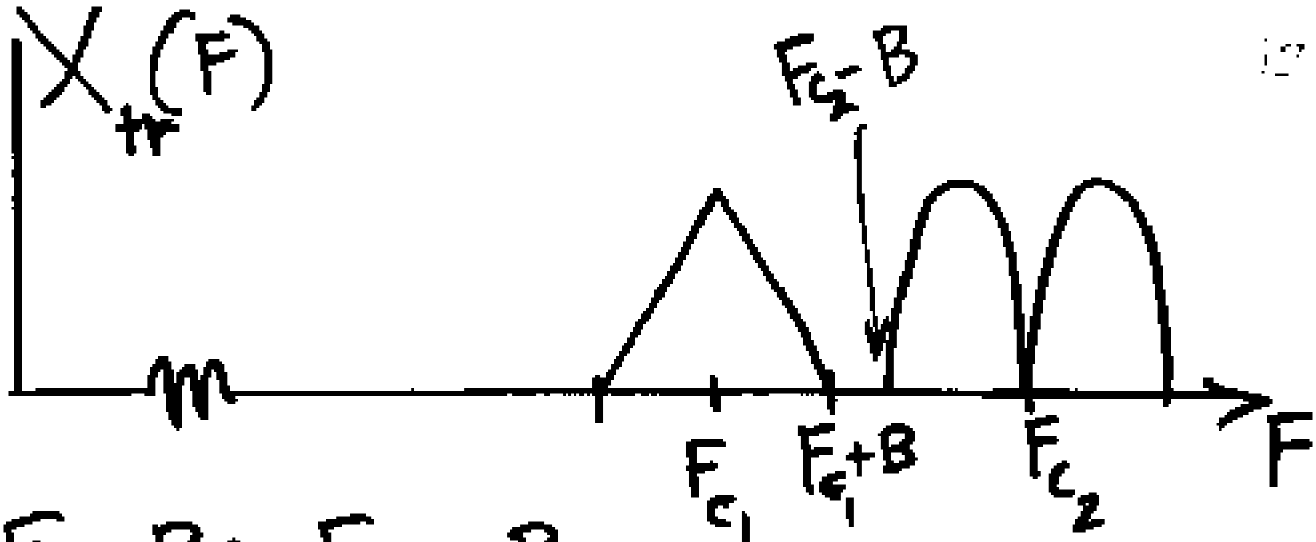


$$\omega_c = \min\left\{\frac{\pi}{L}, \frac{\pi}{D}\right\}$$

- Digital Subbanding
- Preamble: Analog Frequency Division Multiplexing
- transmit multiple signals simultaneously by placing them in different frequency bands

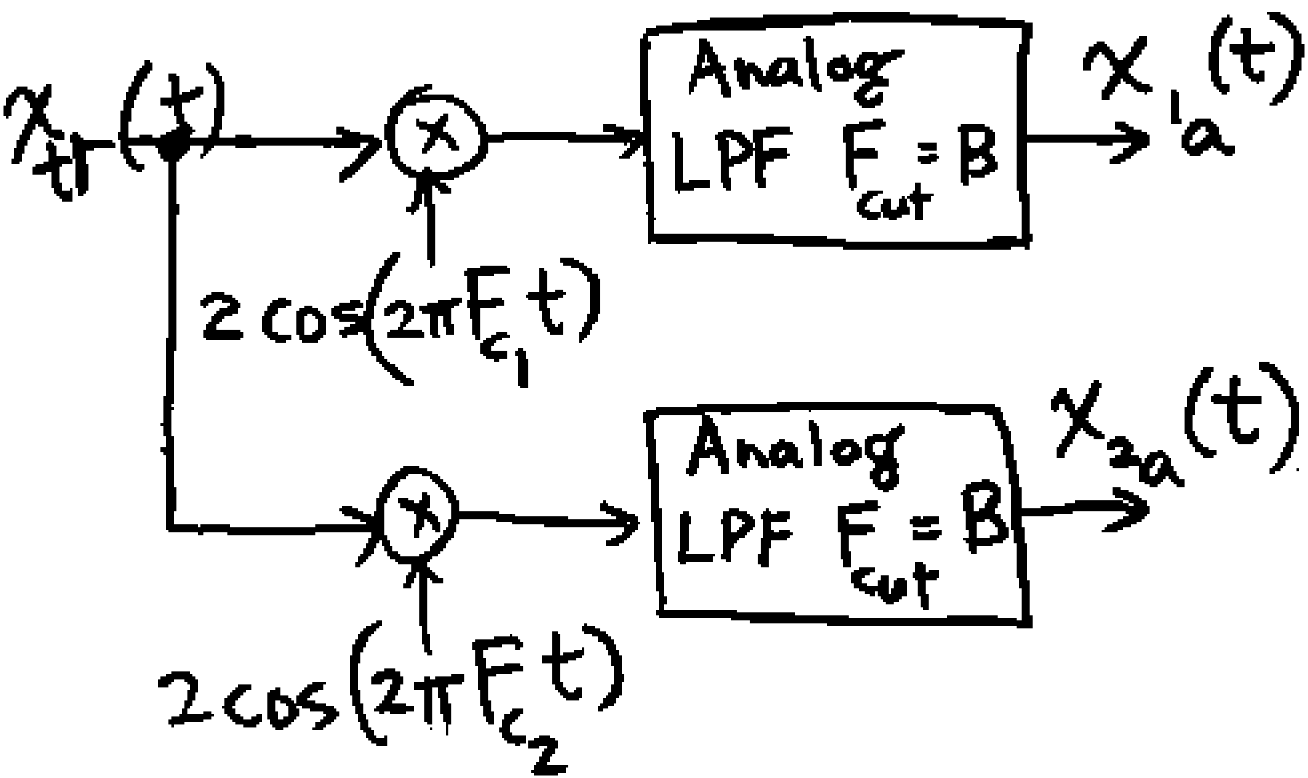


$$\boxed{F_{c_2} - F_{c_1} \geq 2B}$$



$$F_{c_2} - B > F_{c_1} + B$$

• at receiver:



$x_{tr}(t)$
$\times$
Analog
LPF $F_{cut} = B$
$x'_a(t)$
$2\cos(2\pi F_{c_1} t)$
$\times$
Analog
LPF $F_{cut} = B$
$x_{2a}(t)$
$2\cos(2\pi F_{c_2} t)$

- follows from either modulation property of CTFT or

$$\cos^2(2\pi F_c t) = \frac{1}{2} + \frac{1}{2}\cos\left(2\pi(2F_c)t\right)$$

and

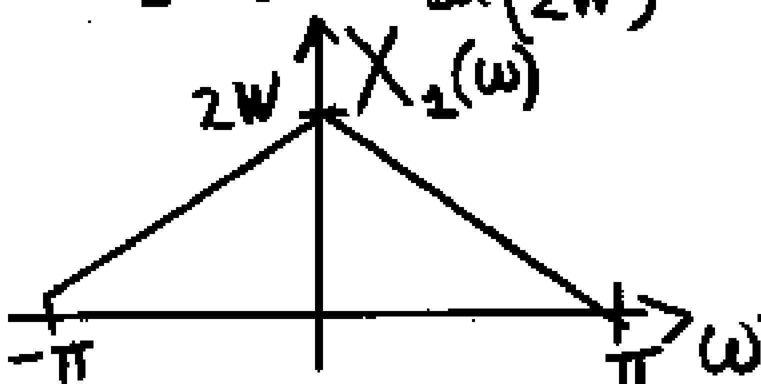
$$\cos(2\pi F_{c_1} t)\cos(2\pi F_{c_2} t)$$

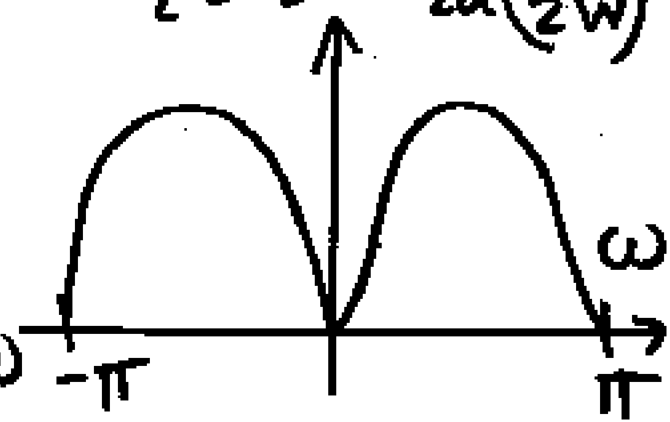
$$= \frac{1}{2}\cos\left[2\pi(F_{c_1} - F_{c_2})t\right]$$

$$+ \frac{1}{2}\cos\left[2\pi(F_{c_1} + F_{c_2})t\right]$$

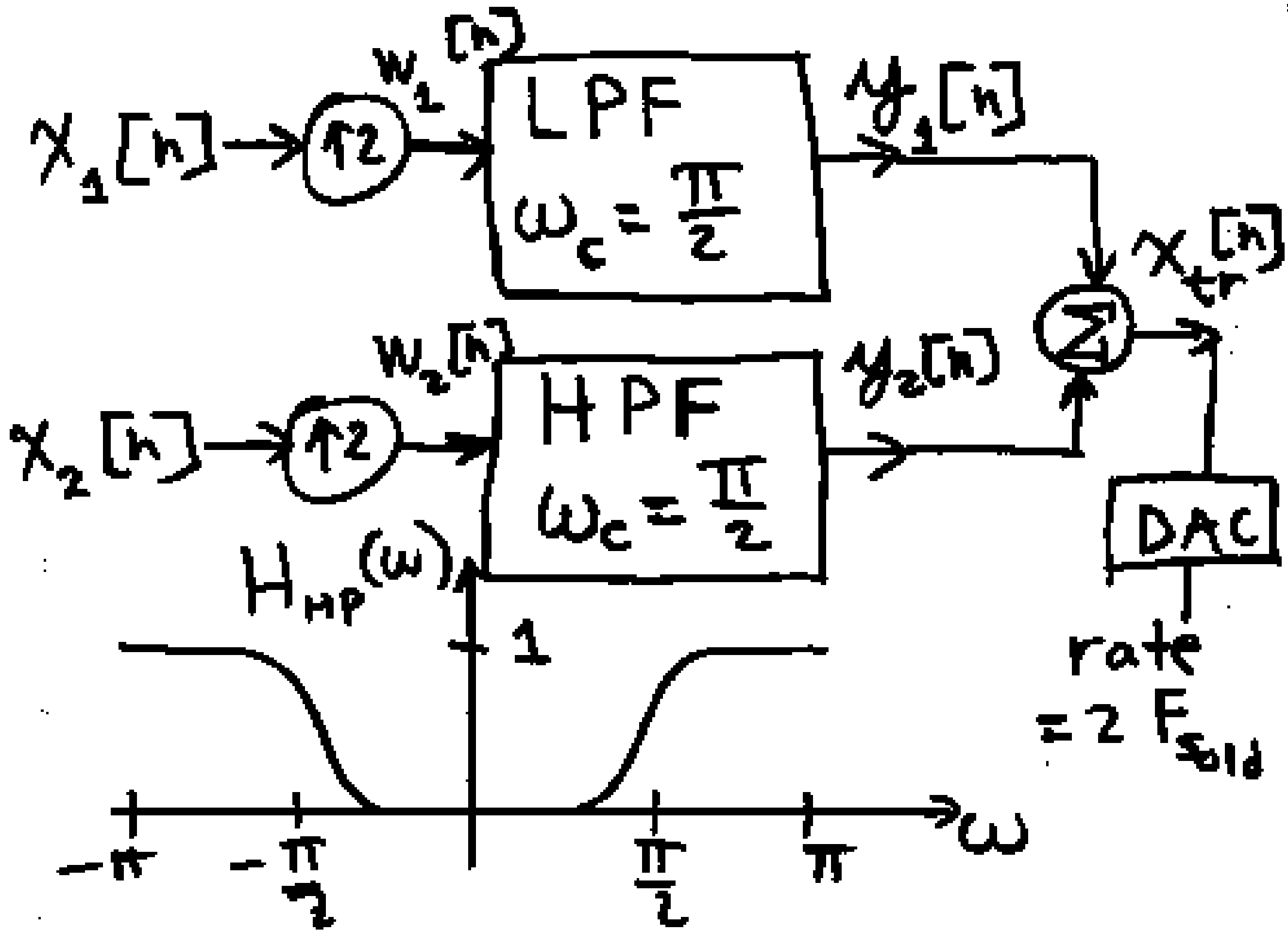
- Digital Subbanding
- Suppose we sample $x_{1a}(t)$ and $x_{2a}(t)$ at/near Nyquist rate

$x_1[n] = x_{1a}\left(\frac{n}{2W}\right)$ $\quad$ $x_2[n] = x_{2a}\left(\frac{n}{2W}\right)$

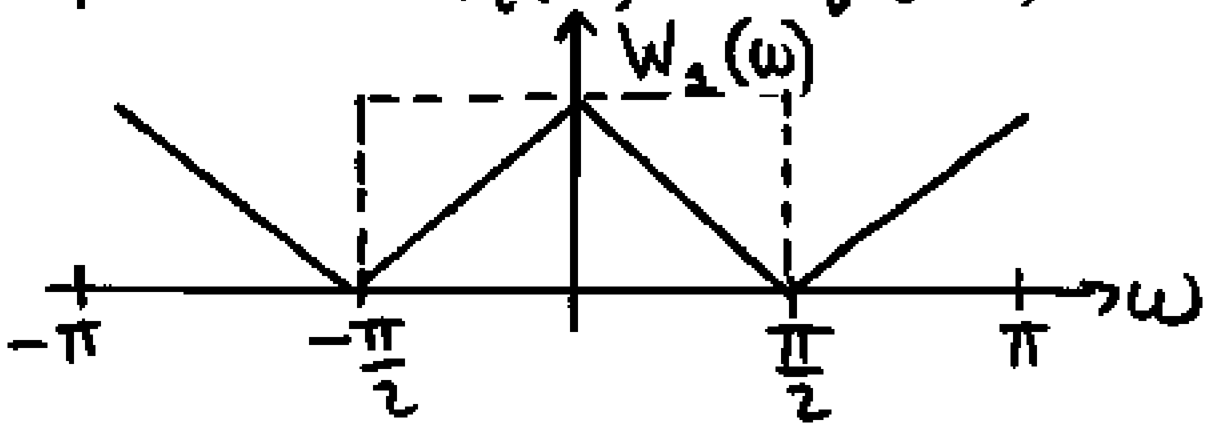


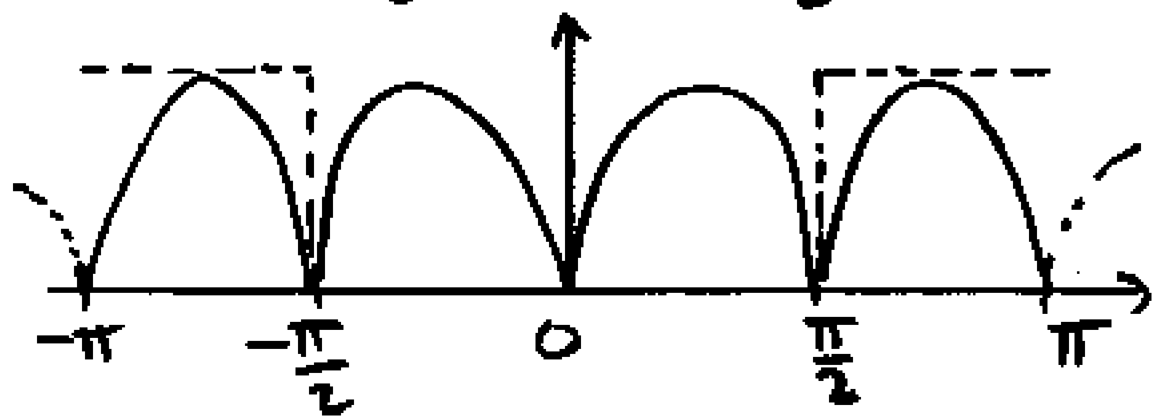


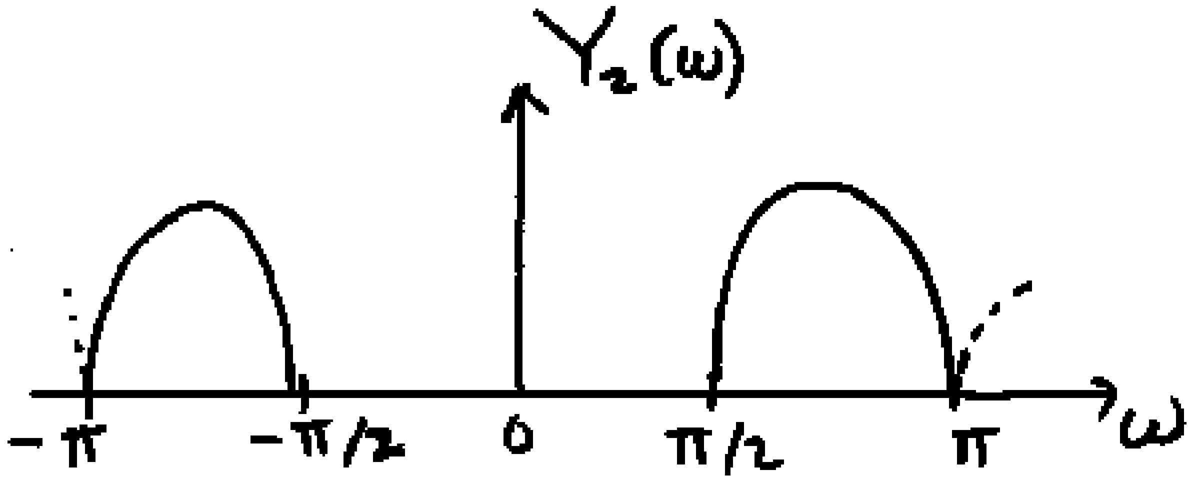
- both signals occupy entire digital frequency band $-\pi < \omega < \pi$ (both periodic @ period 2π)
- How do we place the signals in different subbands
- can't do it by modulation, i.e., multiplying by a cosine
- **Answer:** first do digital upsampling


$x_1[n]$
$\uparrow 2$
$w_1[n]$
LPF
$\omega_c = \frac{\pi}{2}$
$y_1[n]$
$x_2[n]$
$\uparrow 2$
$w_2[n]$
HPF
$\omega_c = \frac{\pi}{2}$
$y_2[n]$
Σ
$x_{tr}[n]$
DAC
rate
$= 2F_{s\,old}$
$H_{HP}(\omega)$
1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
ω

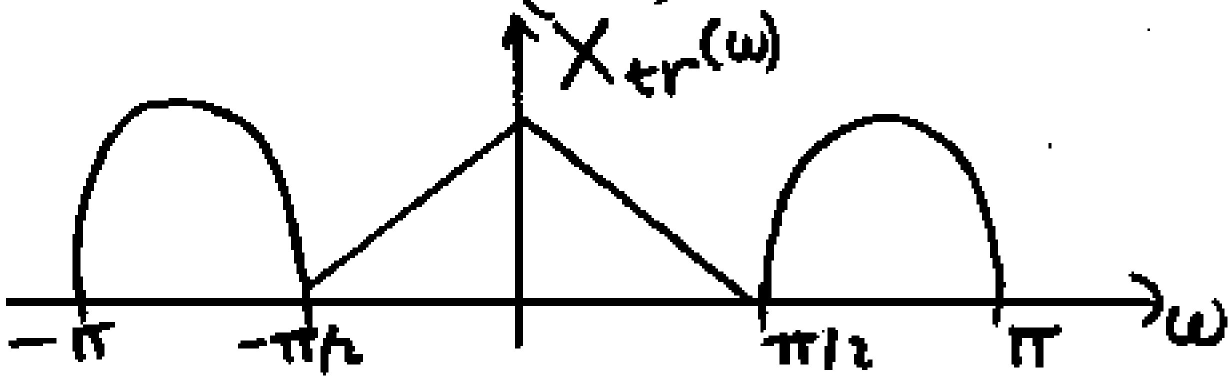
· Recall: $W_i(\omega) = X_i(2\omega)$



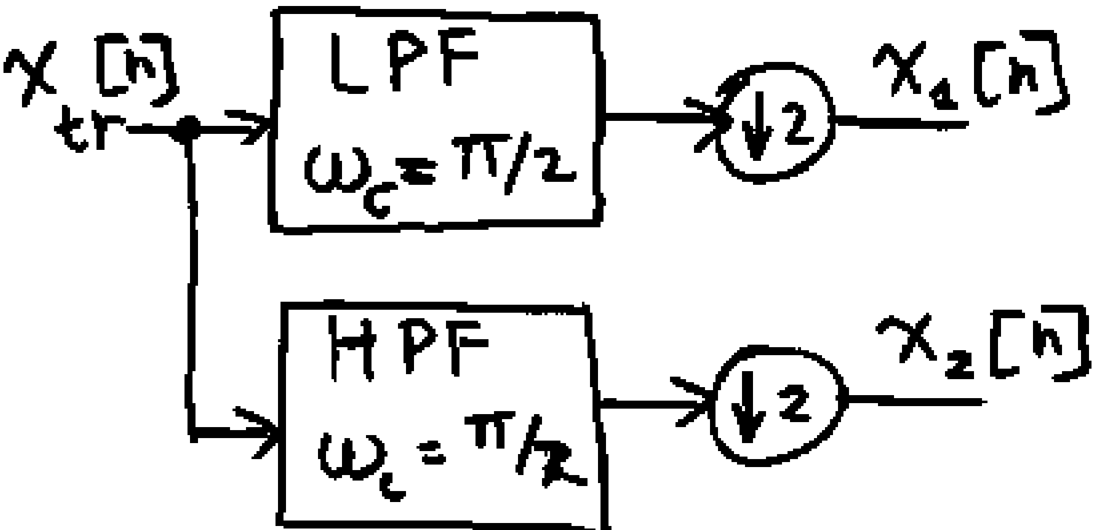




$$y_2[n] = x_{2a}\left(\frac{n}{4W}\right)\cos(\pi n)$$



- How do recover $x_1[n]$ and $x_2[n]$?



$$x_2[n] = y_2[2n]$$

$$= x_{2a}\left(\frac{2n}{4W}\right)\cos(2\pi n) = x_{2a}\left(\frac{n}{2W}\right)$$

$$= x_2[n]$$