EE538 **Final Exam** **Fall 2000**

Digital Signal Processing I **Live: 11 December 2000**

Cover Sheet

Test Duration: 120 minutes.
Open Book but Closed Notes.
Calculators **not** allowed.
This test contains **five** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s) of Problem**	**Points**
1.	Principles of Upsampling and Downsampling	20
2.	Foundation of Adaptive Filtering: MMSE	20
3.	Z-Transform, Interconnection of LTI Systems	20
4.	Sum of Sinewaves Spectral Analysis	20
5.	Digital Subbanding	20

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Problem 1. [20 points]

(a) Find $F(\omega)$ in Figure 1(b) in terms of $H(\omega)$ in Figure 1(a) such that the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

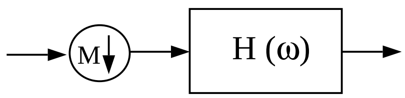

Figure 1(a).

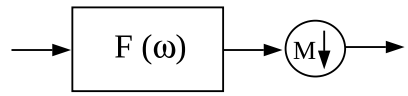

Figure 1(b).

(b) Find $E(\omega)$ in Figure 2(b) in terms of $G(\omega)$ in Figure 2(a) such that the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

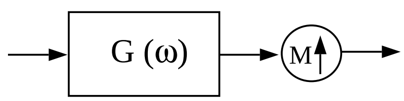

Figure 2(a).

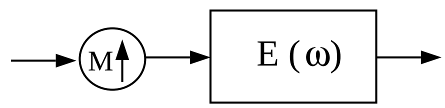

Figure 2(b).

Problem 2. [20 points]
The values of the autocorrelation sequence, $r_{xx}[m] = E\{x[n]x[n-m]\}$, for a (real-valued) discrete-time random process $x[n]$ for two different lag values are

$$r_{xx}[0] = 1, \qquad r_{xx}[1] = \frac{1}{2}.$$

Consider estimating a desired sequence, $d[n]$, as a linear combination of $x[n]$ and $x[n-1]$ according to

$$\hat{d}[n] = w_0 x[n] + w_1 x[n-1]$$

Given the following two values of the cross-correlation, $r_{dx}[m] = E\{d[n]x[n-m]\}$, between $d[n]$ and $x[n]$,

$$r_{dx}[0] = \frac{1}{2}, \qquad r_{dx}[1] = \frac{1}{8}.$$

determine the numerical values of w_0 and w_1 that minimize the mean square error

$$\text{Minimize}_{w_0,w_1} \; E\left\{|d[n] - w_0 x[n] - w_1 x[n-1]|^2\right\}$$

In addition, determine the minumum value of the mean square error. Show all work in arriving at your answer.

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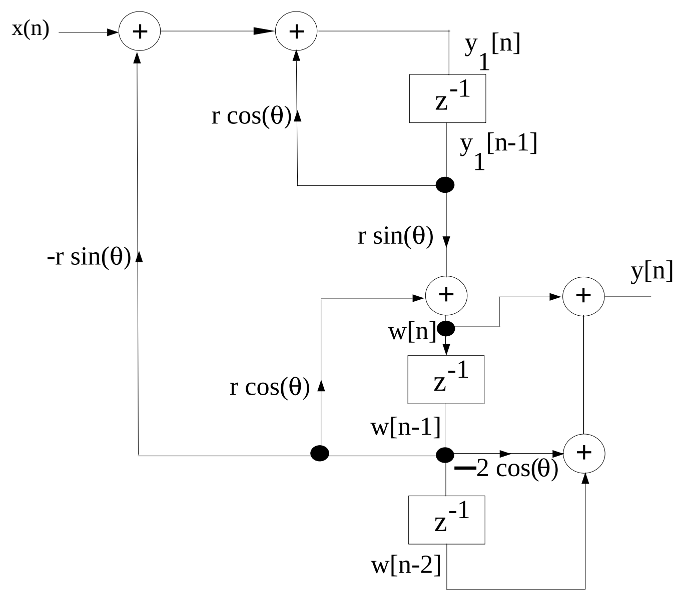
Problem 3. [20 points]

A realization of a causal IIR notch filter is the so-called coupled realization pictured below.



(a) Determine the overall transfer function $H(z) = Y(z)/X(z)$ in terms of r and θ. Show all work. Note that it may be useful to view the overall system as two systems in series and first determine the input/output relationship between $x[n]$ and $w[n]$ followed by determining the input/output relationship between $y[n]$ and $w[n]$.

(b) Express the poles of the systems in terms of r and θ. What are the conditions on r and θ in order that the system be BIBO stable?

(c) For this part of the problem, let $r = .95$ and $\theta = 90°$.

(i) Plot the pole-zero diagram.

(ii) State and plot the region of convergence for $H(z)$.

(iii) Determine the DTFT of $h[n]$ and plot the magnitude $|H(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H(\omega)|$ is exactly zero.

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Problem 4. [20 points]

Consider the discrete-time random process

$$x[n] = D + A\ \cos(2\pi f_o n + \Theta) + \nu[n]$$

where the DC offset, D, and the amplitude, A, and frequency, f_o, of the sinusoid are each deterministic but unknown constants, Θ is a random variable uniformly distributed over a 2π interval, and $\nu[n]$ is a (real-valued) stationary random process with zero mean and $r_{\nu\nu}[m] = E\{\nu[n]\nu[n-m]\} = 2\delta[m]$. That is, $\nu[n]$ forms an i.i.d. sequence with a variance of 2. Note, $\nu[n]$ is independent of Θ for all n.

The values of the autocorrelation sequence for $x[n]$, $r_{xx}[m] = E\{x[n]x[n-m]\}$, for four different lag values are given below.

$$r_{xx}[0] = 8, \quad r_{xx}[1] = 4, \quad r_{xx}[2] = 2, \quad r_{xx}[3] = 4.$$

Determine the numerical values of D, A, and f_o. Show all work.

Problem 5. [20 points]

Let $x_{a1}(t)$, $x_{a2}(t)$, $x_{a3}(t)$, and $x_{a4}(t)$ be four real-valued (lowpass) signals having the same bandwidth, B, and with corresponding CTFT's $X_{a1}(F)$, $X_{a2}(F)$, $X_{a3}(F)$, and $X_{a4}(F)$ depicted in the block diagram on the next page, Figure 4. Each signal is sampled at the Nyquist rate of $F_s = 2B$. The three signals are processed and subsequently summed as shown in Figure 4. Defining $h_{LP}[n]$ as

$$h_{LP}(n) = \frac{\sin(\frac{\pi}{2}n)}{\pi n}, \qquad -\infty < n < \infty,$$

$\tilde{h}_{LP}[n]$ is the (ideal) Discrete-Time Hilbert Transform of $h_{LP}[n]$. (If necessary, see pages 657-659 of the text for both a frequency and time-domain description of the ideal DT Hilbert Transform.)

As indicated in Figure 4 on the next page, $h_1[n] = h_{LP}[n] + j\tilde{h}_{LP}[n]$. The respective impulse responses of each of the other three filters are $h_2[n] = e^{j\frac{\pi}{2}n}h_1[n]$, $h_3[n] = e^{-j\frac{\pi}{2}n}h_1[n]$, $h_4[n] = e^{-j\pi n}h_1[n]$.

(a) Let $Y(\omega)$ denote the DTFT of the sum signal, $y[n]$, at the output. Plot the magnitude of $Y(\omega)$ over $-\pi < \omega < \pi$. Show as much detail as possible. *You do NOT need to show a lot of work in arriving at your answer. If you know what the system is doing, draw your answer and provide a brief explanation. Think about what's happening in the frequency domain – don't even think about doing any convolution.*

(b) Draw a block diagram of a system for recovering each of the four original sampled signals, $x_1[n]$, $x_2[n]$, $x_3[n]$, and $x_4[n]$, from the sum signal, $y[n]$. **In contrast to Exam 2, for this exam you have to provide a computationally efficient scheme for recovering each of the signals. You can ONLY use an interconnection of LTI filters to recover each signal. Specifically, you cannot use any of the following sub-systems:**

  (i) You **cannot** use decimators. (Recall that filtering followed by decimation is inefficient since you throw away values of the computed filter output.)

  (ii) You **cannot** take the real part of a signal. (That is also inefficient since that would imply that you computed the imaginary part of an output only to ultimately throw it away.)

  (iii) You **cannot** use any kind of modulation, that is, multiplication by a sinewave is **not** permitted.

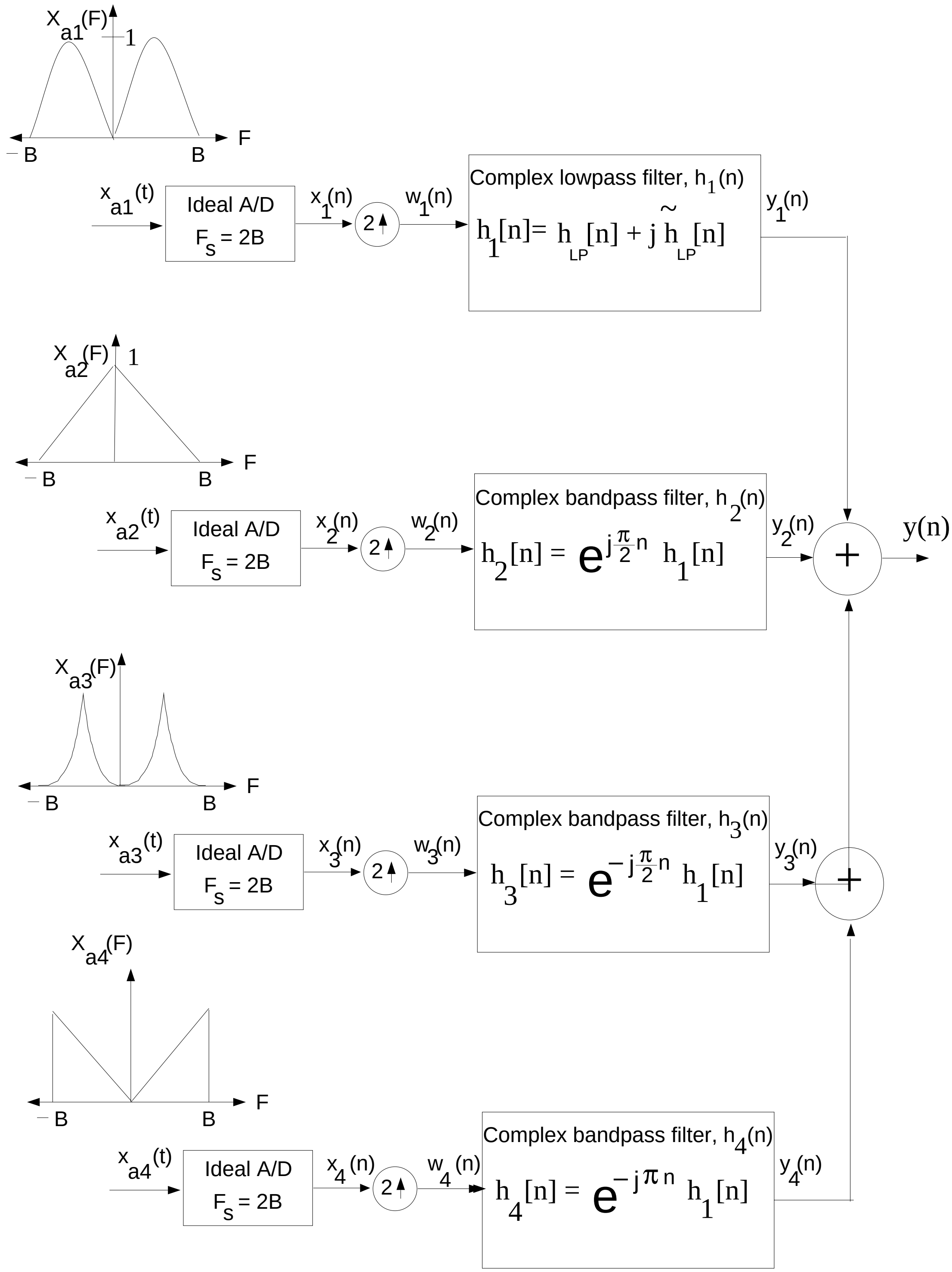

Figure 4: Digital subbanding of four real-valued signals each sampled at Nyquist rate.