EE538 (NTU: CC-560-M) Final Exam Fall 1998
Digital Signal Processing I Live: 14 December 1998

Cover Sheet

Test Duration: 120 minutes.
Open Book but Closed Notes.
Calculators allowed.
This test contains **six** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s) of Problem**	**Points**
1.	Difference Equations, LTI Systems	30
2.	Sampling, Z-Transform, ZT-DTFT Relationship	30
3.	DFT and Time-Domain Aliasing	30
4.	Digital Upsampling and Downsampling	30
5.	Autoregressive Spectral Analysis	40
6.	Windows & Symmetric FIR Filters	40

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Problem 1. [30 points]
Consider the causal LTI system implemented as

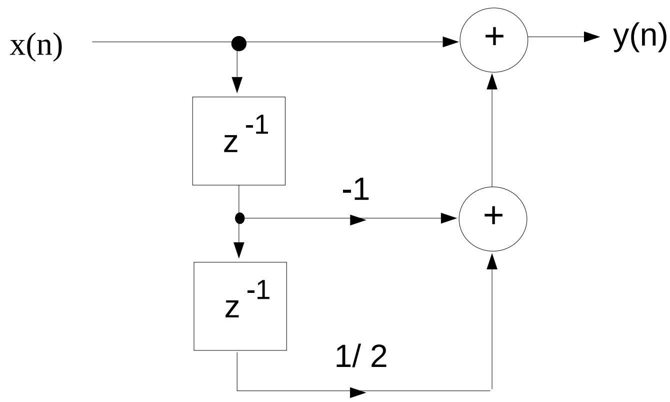

Figure P1.1

(a) Determine the difference the difference equation relating the output, $y[n]$, to the input, $x[n]$.

The DT system in Figure P1.1 may be alternatively implemented via a lattice structure as

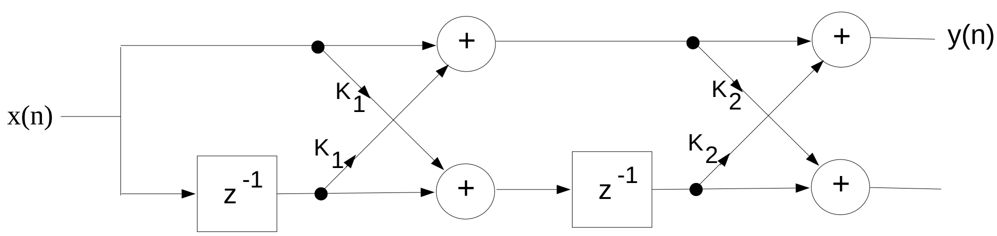

Figure P1.2

(b) For the system in Figure P1.2, determine the difference equation relating the output, $y[n]$, to the input, $x[n]$.

(c) Determine the numerical values of K_1 and K_2 so that the relationship between $x[n]$ and $y[n]$ in Figure P1.2 is exactly the same as the relationship between $x[n]$ and $y[n]$ in Figure P1.1, so that the respective difference equations are identical.

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Problem 2. [30 points]
Consider the analog signal

$$x_a(t) = e^{-(4\ln 2)t}\cos(2\pi t)u(t)$$

where $\ln x$ denotes the natural logarithm of x such that $e^{\ln x} = x$ (Hint: the numbers are selected to work out "nicely".) $x_a(t)$, which may be viewed as a transient in an analog circuit, is sampled at a rate $F_s = 8$ Hz to produce the discrete-time signal $x[n]$, i.e.,

$$x[n] = x_a(nT_s) \qquad T_s = \frac{1}{8} \text{ second}$$

(a) Determine the Z-transform, $X(z)$, of $x[n]$ and state the region of convergence. (You may use the tables in the book.)

(b) Determine the DTFT of $x[n]$, $X(\omega)$, and provide a rough sketch of $|X(\omega)|$ over $|\omega| < \pi$.

Problem 3. [30 points]
Let $x[n]$ and $h[n]$ be two finite duration sequences of length $L = 6$, i.e., $x[n] = h[n] = 0$ for $n < 0$ and $n \geq 6$. Let $X_6(k)$ and $H_6(k)$ denote 6-point DFT's of $x[n]$ and h[n], respectively. The 6-point inverse DFT of the product $Y_6(k) = X_6(k)H_6(k)$, denoted $y_6[n]$, produces the following values:

n	0	1	2	3	4	5
$y_6[n]$	21	21	21	21	21	21

Let $X_9(k)$ and $H_9(k)$ denote the 9-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 9-point inverse DFT of the product $Y_9(k) = X_9(k)H_9(k)$, denoted $y_9[n]$, produces the following values:

n	0	1	2	3	4	5	6	7	8
$y_9[n]$	9	12	15	18	20	21	15	10	6

Given $y_6[n]$ and $y_9[n]$, find the **linear** convolution of $x[n]$ and $h[n]$, i. e., determine the numerical values of $y[n] = x[n] * h[n]$.

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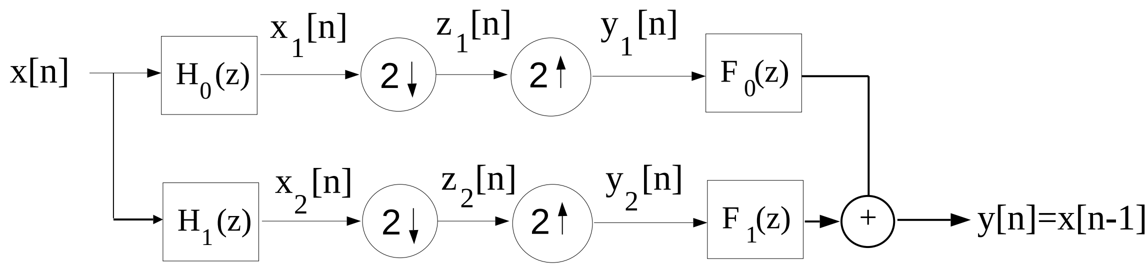
Problem 4. [30 points]

In the figure below, $H_0(z) = 1 + z^{-1}$ and $H_1(z) = 1 - z^{-1}$. Determine the Z-Transform of the synthesis filters, $F_0(z)$ and $F_1(z)$ so that $y[n] = x[n-1]$, i.e., that the output of the overall system is equal to the input delayed by one time unit for any and all possible input sequences. Show all work in arriving at your answers.



In the figure above,

$$z_i[n] = x_i[2n], \quad i = 1, 2$$

and

$$y_i[n] = \begin{cases} z_i(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}, \quad i = 1, 2$$

Problem 5. [40 points]

The autocorrelation values for an autoregressive process of order $p = 2$ (AR(2) process) for lag values $m = 0$, $m = 1$, and $m = 2$ are

$$r_{xx}[0] = 1 \quad r_{xx}[1] = \tfrac{1}{2} \quad r_{xx}[2] = \tfrac{1}{8}$$

(a) Determine the AR model parameters a_1 and a_2 and the power, σ_w^2, of the input white noise process that generated the AR process.

(b) Determine the numerical values of $r_{xx}[3]$ and $r_{xx}[4]$.

(c) Determine the DTFT of $r_{xx}[m]$ defined as

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}(m) e^{-j\omega m}$$

Note: There is no need to compute a DTFT here; just determine $S_{xx}(\omega)$ but note infinite limits, not finite limits in the sum.

Problem 6. [40 points]
Consider the (causal) window of length M described as

$$w[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right), & 0 \le n \le \frac{M}{4} - 1 \\ 1, & \frac{M}{4} \le n \le \frac{3M}{4} - 1 \\ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right), & \frac{3M}{4} \le n \le M - 1 \end{cases}$$

where M is divisible by 4, *i.e.*, that $\frac{M}{4}$ is an integer. $w[n]$ may also be expressed as

$$w[n] = \left\{\frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right)\right\}\{u[n] - u(n-M)\}$$

$$+\left\{\frac{1}{2} + \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right)\right\}\left\{u\left(n - \frac{M}{4}\right) - u\left(n - \frac{3M}{4}\right)\right\}$$

(a) Plot a rough sketch of the window as a function of n. Is the window symmetric or anti-symmetric, *i.e.*, $w[n] = w(M-1-n)$ or $w[n] = -w(M-1-n), n = 0, 1, ..., M-1$?

In contrast to the window problem on Exam 3, your **brief** answers to the questions below may be explained heuristicly via tapering arguments **without** the need for mathematical analysis, i.e., supporting mathematical analysis is **NOT** necessary.

(b) Is the width of the mainlobe of the DTFT of $w[n]$ wider or narrower than the width of the mainlobe of the DTFT of a rectangular window of the same length. Explain.

(c) Is the width of the mainlobe of the DTFT of $w[n]$ wider or narrower than the width of the mainlobe of of the DTFT of a Hamming window of the same length. Explain.

(d) Is the peak sidelobe level of the DTFT of $w[n]$ larger or smaller than the peak sidelobe level of the DTFT of a rectangular window of the same length. Explain.

(e) Is the peak sidelobe level of the DTFT of $w[n]$ larger or smaller than the peak sidelobe level of the DTFT of a Hamming window of the same length. Explain.

(f) The DTFT of $w[n]$ may be expressed as

$$W(\omega) = e^{-j\frac{M-1}{2}\omega}\left\{\sum_{i=1}^{6} A_i \frac{\sin\left(\frac{L_i}{2}(\omega - \omega_i)\right)}{\sin\left(\frac{1}{2}(\omega - \omega_i)\right)}\right\}$$

Determine the values of A_i, L_i, and ω_i, $i = 1, ..., 6$. The answers for A_i, $i = 1, ..., 6$, are real-valued numbers while the answers for L_i and ω_i, $i = 1, ..., 6$, are in terms of M. List your answers in the form of a table as shown below. *Note:* Answers need to be substantiated by work – no points for pure guesses or recalling from memory.

i	A_i	L_i	ω_i
1			
2			
3			
4			
5			
6			