EE538 Digital Signal Processing I Exam 2 Session 29

Test Duration: 50 minutes. Open Book but Closed Notes.

Do all work in blue books provided. Only return the blue books.

Problem 1. [40 points]

Let $x_{a1}(t)$, $x_{a2}(t)$, $x_{a3}(t)$, and $x_{a4}(t)$ be four real-valued (lowpass) signals having the same bandwidth, B, and with corresponding CTFT's $X_{a1}(F)$, $X_{a2}(F)$, $X_{a3}(F)$, and $X_{a4}(F)$ depicted in Figure 1 on the next page. Each signal is sampled at the Nyquist rate of $F_s = 2B$. The three signals are processed and subsequently summed as shown in Figure 1. Defining $h_{LP}[n]$ as

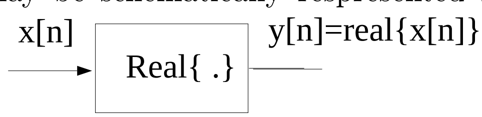
$$h_{LP}(n) = \frac{\sin(\frac{\pi}{2}n)}{\pi n}, \qquad -\infty < n < \infty,$$

$\tilde{h}_{LP}[n]$ is the (ideal) Discrete-Time Hilbert Transform of $h_{LP}[n]$. (If necessary, see pages 657-659 of the text for both a frequency and time-domain description of the ideal DT Hilbert Transform.)

As indicated in Figure 1 on the next page, $h_1[n] = h_{LP}[n] + j\tilde{h}_{LP}[n]$. The respective impulse responses of each of the other three filters are $h_2[n] = e^{j\frac{\pi}{2}n}h_1[n]$, $h_3[n] = e^{-j\frac{\pi}{2}n}h_1[n]$, $h_4[n] = e^{-j\pi n}h_1[n]$.

(a) Let $Y(\omega)$ denote the DTFT of the sum signal, $y[n]$, at the output. Plot the magnitude of $Y(\omega)$ over $-\pi < \omega < \pi$. Show as much detail as possible. *You do NOT need to show a lot of work in arriving at your answer. If you know what the system is doing, draw your answer and provide a brief explanation. Think about what's happening in the frequency domain – don't even think about doing any convolution.*

(b) Draw a block diagram of a system for recovering each of the four original sampled signals, $x_1[n]$, $x_2[n]$, $x_3[n]$, and $x_4[n]$, from the sum signal, $y[n]$. Note that taking the real part of a signal may be schematically respresented in a block diagram as



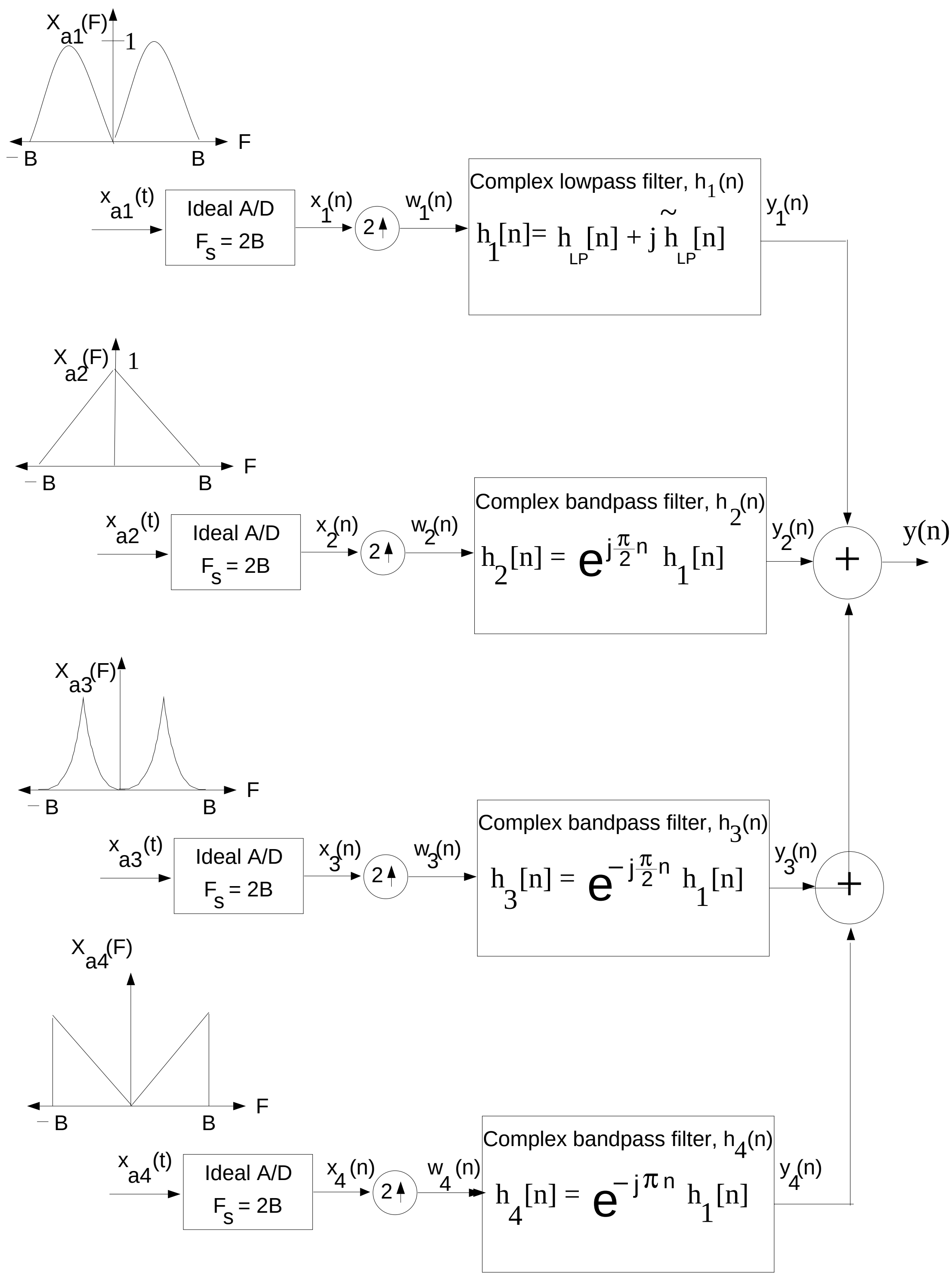


Figure 1: Digital subbanding of four real-valued signals each sampled at Nyquist rate.

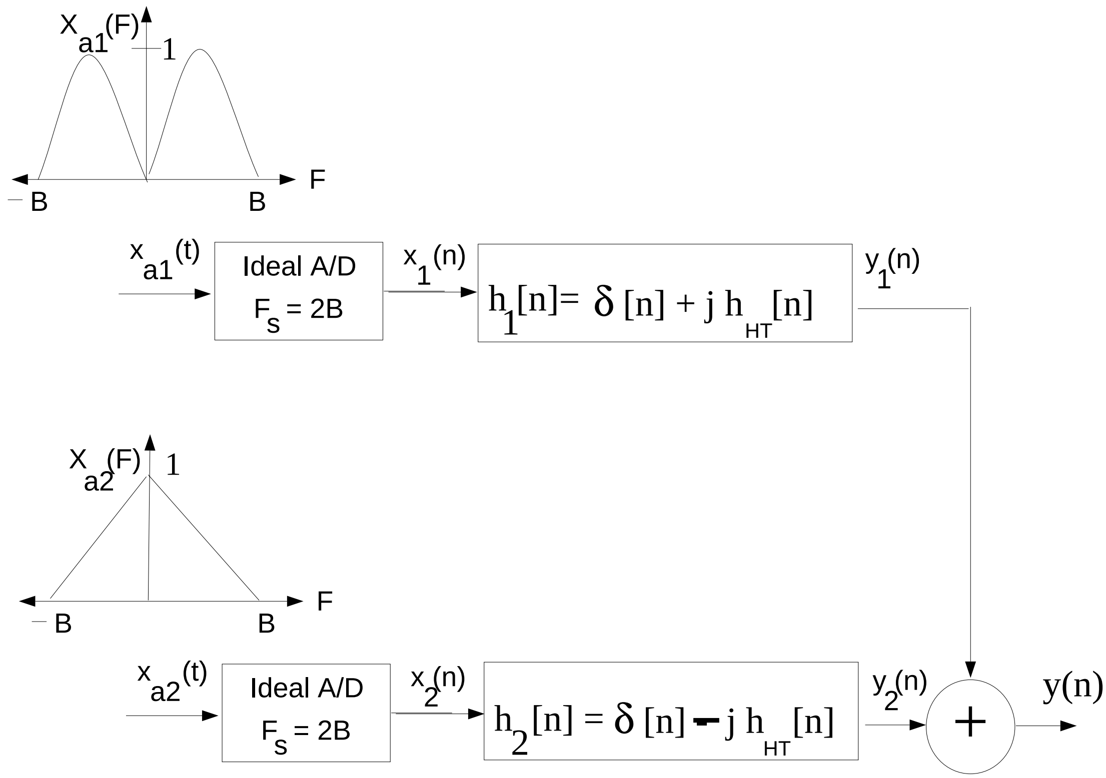


Figure 2: Problem on Hilbert Transform.

Problem 2. [30 points]

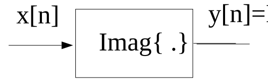
Let $x_{a1}(t)$ and $x_{a2}(t)$ be two real-valued (lowpass) signals having the same bandwidth, B, and with corresponding CTFT's $X_{a1}(F)$ and $X_{a2}(F)$ depicted in Figure 2. Each signal is sampled at the Nyquist rate of $F_s = 2B$. The two signals are processed and subsequently summed as shown in Figure 1. The impulse response $h_{HT}[n]$ is the impulse response of an ideal Hilbert Transformer given on page 659 of the textbook repeated here

$$h_{HT}(n) = 2\frac{\sin^2(\frac{\pi}{2}n)}{\pi n}, \qquad -\infty < n < \infty,$$

As indicated in Figure 2, $h_1[n] = \delta[n] + jh_{HT}[n]$ and $h_2[n] = \delta[n] - jh_{HT}[n]$.

(a) Let $Y(\omega)$ denote the DTFT of the sum signal, $y[n]$, at the output. Plot the magnitude of $Y(\omega)$ over $-\pi < \omega < \pi$. Show as much detail as possible. *You do NOT need to show a lot of work in arriving at your answer. If you know what the system is doing, draw your answer and provide a brief explanation. Think about what's happening in the frequency domain – don't even think about doing any convolution.*

(b) Draw a block diagram of a system for recovering each of the two original sampled signals, $x_1[n]$ and $x_2[n]$, from the sum signal, $y[n]$. Note that taking the imaginary part of a signal may be schematically respresented in a block diagram as



Problem 3. [30 points]

An analog Butterworth filter of order $N = 1$ with a 3-dB cut-off at $\Omega_c = 1$ has the following transfer function (Laplace Transform):

$$H_a(s) = \frac{1}{s+1} \tag{1}$$

A digital filter is synthesized via the following bilinear transformation *which is different from the one used in class or the textbook.*

$$s = \frac{z+1}{z-1} \tag{2}$$

(a) Determine the transfer function, $H(z)$, of the digital filter obtained by applying the bilinear transform in (2) to the analog filter described by (1). Plot the magnitude of the frequency response of the resulting digitial filter over $-\pi < \omega < \pi$ showing as much detail as possible.

(b) Is the resulting digital filter stable?

(c) Determine the difference equation for implementing the resulting digital lowpass filter.