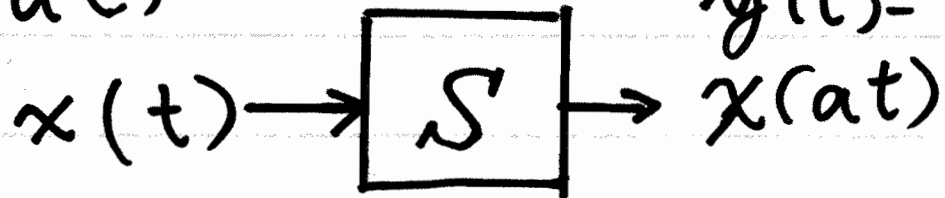


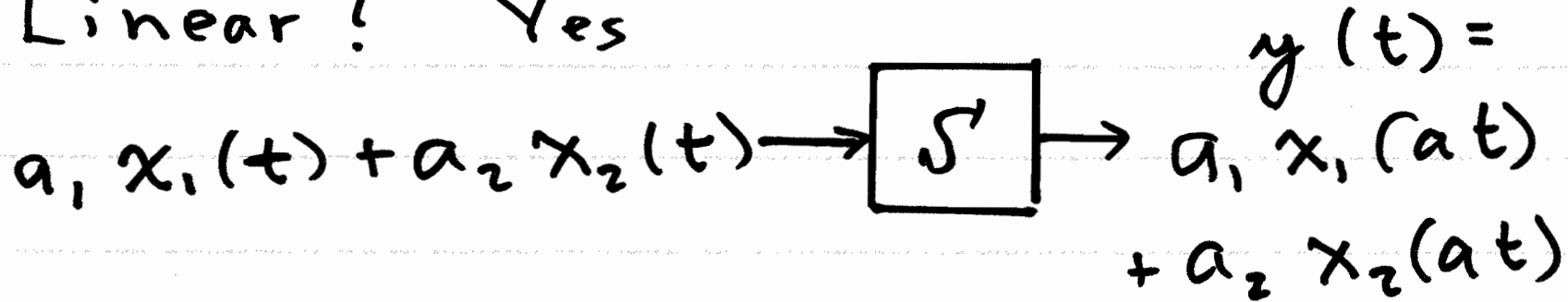
# System Examples

①

1.  $y(t) = x(at)$

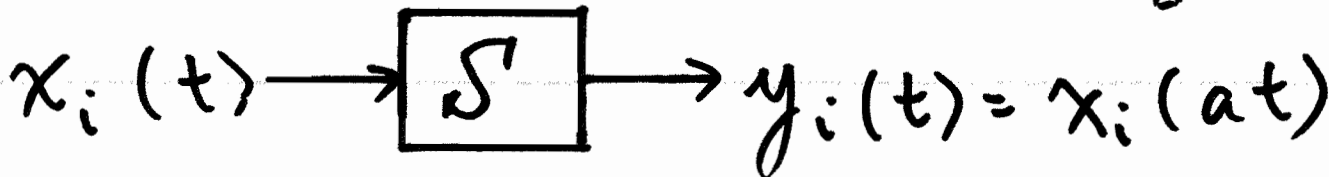


Linear? Yes



where:

$$= a_1 y_1(t) + a_2 y_2(t)$$



$i = 1, 2$

TI? No, not time-invariant (2)

$$x(t) \rightarrow \boxed{\mathcal{S}} \rightarrow y(t) = x(at)$$

$$x(t-t_0) \rightarrow \boxed{\mathcal{S}} \rightarrow z(t) = x(at-t_0)$$

Is  $z(t) \stackrel{?}{=} y(t-t_0)$ ? No

$$x(at-t_0) \neq x(a(t-t_0)) = x(at-at_0)$$

See Example 1.16 in text on pg. 52

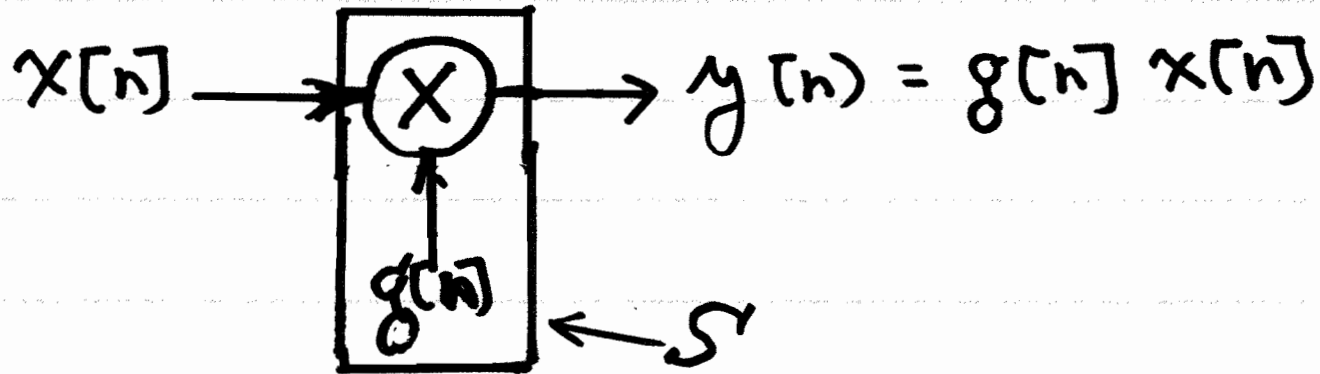
System is stable.

System is not causal if  $a < 0$

Same for  
DT system  
 $y[n] = x[an]$   
 $a$ , integer

## Example 2. DT System (3)

$$y[n] = g[n] x[n]$$



Linear? Yes, due to distributive property of multiplication

$$\begin{aligned} a_1 x_1[n] + a_2 x_2[n] &\rightarrow \boxed{S} \rightarrow y[n] = \\ &\rightarrow g[n] \{ a_1 x_1[n] + a_2 x_2[n] \} \\ &= a_1 g[n] x_1[n] + a_2 g[n] x_2[n] \\ &= a_1 y_1[n] + a_2 y_2[n] \end{aligned}$$

④

TI? No, system is not Time-Invariant

$$x[n-n_0] \rightarrow \boxed{S} \rightarrow z[n] = g[n] x[n-n_0]$$

Is  $z[n] \stackrel{?}{=} y[n-n_0]$ ?  $y[n] = g[n] x[n]$

↓

$$g[n] x[n-n_0] \neq g[n-n_0] x[n-n_0]$$

System is not TI

System is BIBO Stable if  $|g[n]| < \infty \forall n$

System is causal (it is memoryless)

Examples:  $g[n] = \cos(\omega_0 n)$

$$g[n] = n$$

See Hmwk. 2

## Comments:

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- Similar observations hold with respect to the CT System  $y(t) = g(t)x(t)$ 
  - See Eg. 1.17 pg. 54 where  $g(t) = t$
- Even though this system is not both Linear and Time-Invariant (LTI), it is still a very useful system in practice
- The only ramification of not being LTI is that the output is not related to the input through convolution with the impulse response of the system = but still useful

### Example 3. Square-Law System (6)

$$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x^2(t)$$

Linear? No. Superposition does not apply.

$$a_1 x_1(t) + a_2 x_2(t) \rightarrow \boxed{S} \rightarrow y(t) \Rightarrow$$

$$\Rightarrow \{a_1 x_1(t) + a_2 x_2(t)\}^2 =$$

$$= a_1^2 x_1^2(t) + \underbrace{a_1 a_2 x_1(t) x_2(t)}_{\text{violates linearity}} + a_2^2 x_2^2(t)$$

↑  
violates  
linearity

↑  
violates  
linearity

↑  
violates  
linearity

$$\neq a_1 x_1^2(t) + a_2 x_2^2(t)$$

• See Example 1.18 in text on pg. 54 ⑦

• System is TI:  $x(t-t_0) \rightarrow \boxed{\int} \rightarrow x^2(t-t_0)$

• stable and causal too

• Similar observations hold wrt DT system

$$y[n] = x^2[n]$$

• Again, even though not LTI, this system is used often in practice, as we will see later in this course

• See handout posted at course website for further examples of systems

## Example: Hard-Limiter

(8)

$$y(t) = \begin{cases} V_{\max}, & \text{if } x(t) > V_{\max} \\ x(t), & \text{if } -V_{\max} < x(t) < V_{\max} \\ -V_{\max}, & \text{if } x(t) < -V_{\max} \end{cases}$$

- used to remove noisy spikes in radios (FM)
- used, after <sup>amplitude-</sup>scaling to fit signal into range of quantizer in process of A/D conversion
- System is TI but not linear
- You can show nonlinearity with a single counter-example  $\Rightarrow x_1(t) = A_1 u(t)$  and  $x_2(t) = A_2 u(t)$ , where  $A_1 < V_{\max}$  and  $A_2 < V_{\max}$  BUT  $A_1 + A_2 > V_{\max}$



$$x_1(t) \rightarrow \boxed{S} \rightarrow y_1(t) = A_1 u(t)$$

$$x_2(t) \rightarrow \boxed{S} \rightarrow y_2(t) = A_2 u(t)$$

$$x_1(t) + x_2(t) \rightarrow \boxed{S} \rightarrow y(t) = \sqrt{\max} u(t)$$

$$\neq y_1(t) + y_2(t)$$

$$= (A_1 + A_2) u(t)$$

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## Example. Integrator

10

$$y(t) = \int_{t-T_b}^{t+T_a} x(\tau) d\tau = \left\{ \begin{array}{l} \text{area under} \\ x(t) \\ \text{from } t-T_b \\ \text{to } t+T_a \end{array} \right.$$

$T_a > 0, T_b > 0$ . note:  $T_b = \infty$  is valid

System is LTI.

Linearity:

$$y(t) = \int_{t-T_b}^{t+T_a} (a_1 x_1(\tau) + a_2 x_2(\tau)) d\tau$$

$$= a_1 \int_{t-T_b}^{t+T_a} x_1(\tau) d\tau + a_2 \int_{t-T_b}^{t+T_a} x_2(\tau) d\tau$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

TI: input  $x(t-t_0)$ :

(11)

$$z(t) = \int_{t-T_b}^{t+T_a} x(\tau-t_0) d\tau$$

• change of variables:  $\lambda = \tau - t_0$

$$d\lambda = d\tau \quad \tau = \lambda + t_0$$

new limits:

$$\lambda \left[ \begin{array}{l} t+T_a-t_0 \\ t-T_b-t_0 \end{array} \right.$$

THUS;

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$$z(t) = \int_{t-t_0-T_b}^{t-t_0+T_a} x(\lambda) d\lambda = y(t-t_0)$$

∴ system is LTI