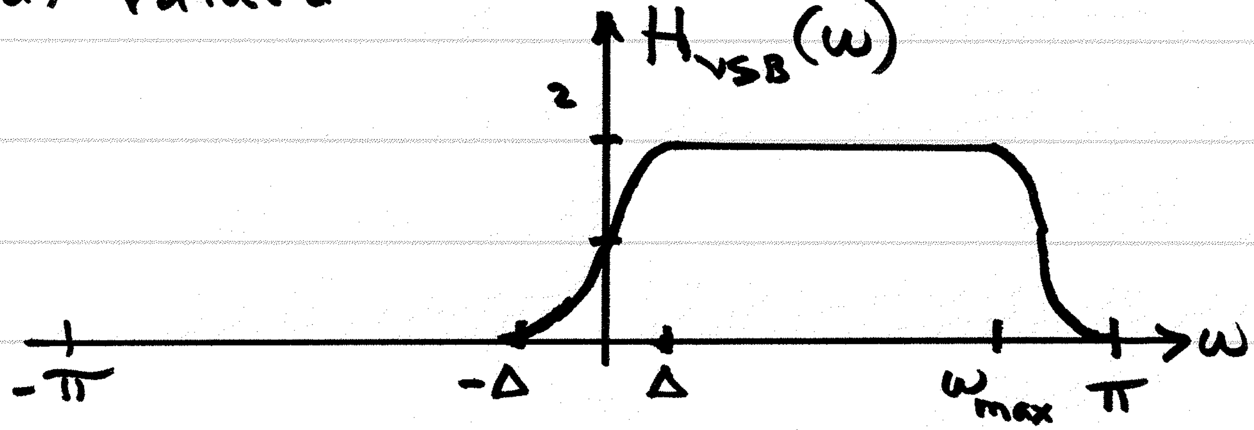
Supplemental Notes on VSB Modulation

$$x[n] \longrightarrow \boxed{h_{VSB}[n]} \longrightarrow x_{VSB}[n]$$

real-valued



$$H_{VSB}(-\omega) + H_{VSB}(\omega) = 2 \quad \text{for } |\omega| < \Delta$$

also: $X(\omega) = 0$ for $\omega_{max} < |\omega| < \pi$ (sample a little bit of above Nyquist rate)

Under these conditions:

$$x_{VSB}[n] = x[n] + j\left\{ x[n] * \operatorname{Im}\{h_{VSB}[n]\} \right\}$$

- **Note:** $h_{VSB}[n]$ is complex-valued but $H_{VSB}(\omega)$ is real-valued
- Proof that $Re\{x_{VSB}[n]\} = x[n]$ if $H_{VSB}(-\omega) + H_{VSB}(\omega) = 2$ for $|\omega| < \omega_{max}$ $\left(= 2\pi \frac{W}{F_s}\right)$

- $$Re\{x_{VSB}[n]\} = \frac{1}{2} x_{VSB}[n] + \frac{1}{2} x^*_{VSB}[n]$$

$$\xleftrightarrow{DTFT} \frac{1}{2} H_{VSB}(\omega) X(\omega) + \frac{1}{2} H^*_{VSB}(-\omega) X^*(-\omega)$$

Since $H_{SVB}(\omega)$ is real-valued, $H^*_{VSB}(-\omega) = H_{VSB}(-\omega)$

Since $x[n]$ is real-valued, $X^*(-\omega) = X(\omega)$

THUS:

$$Re\{x_{VSB}[n]\} \xleftrightarrow{DTFT} \frac{1}{2}\{H_{VSB}(\omega) + H_{VSB}(-\omega)\} X(\omega)$$

$$= \frac{1}{2}(2) X(\omega) = X(\omega)$$

$= x[n]$

- That is, $x[n]$ is the real part of $x_{VSB}[n]$ (2)

- $\Delta = 0$ is just special case of single-sideband where $\mathrm{Im}\{h_{VSB}[n]\} = h_{HT}[n] \Rightarrow$ Hilbert Transformer

- $h_{VSB}[n] = \mathrm{Re}\{h_{VSB}[n]\} + j\,\mathrm{Im}\{h_{VSB}[n]\}$

$$= h^{(r)}_{VSB}[n] + j\,h^{(i)}_{VSB}[n]$$

Define: $\hat{x}[n] = x[n] * h^{(i)}_{VSB}[n]$

Then: $x_{VSB}[n] = x[n] + j\,\hat{x}[n]$

$X_{VSB}(\omega) = 0$ for $-\pi < \omega \leq -\Delta$

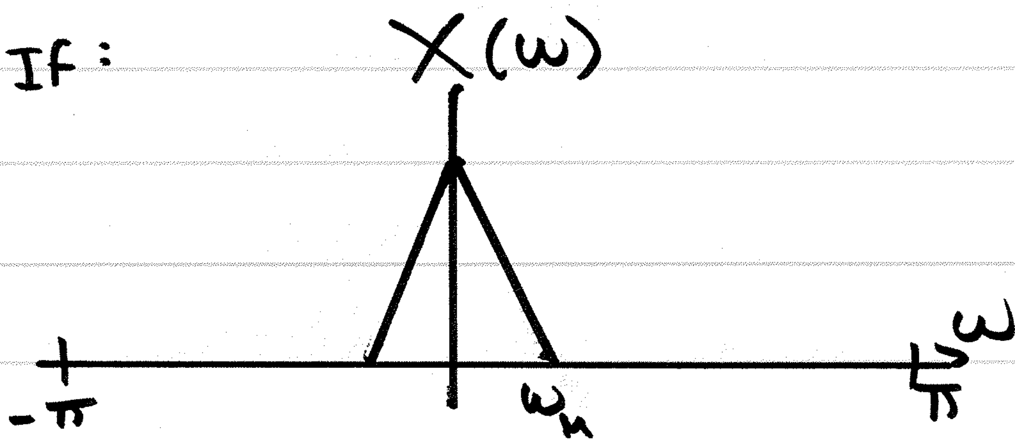
- Multiply by sinewave to put in different frequency band: since we can only transmit real-valued signals, consider: (3)

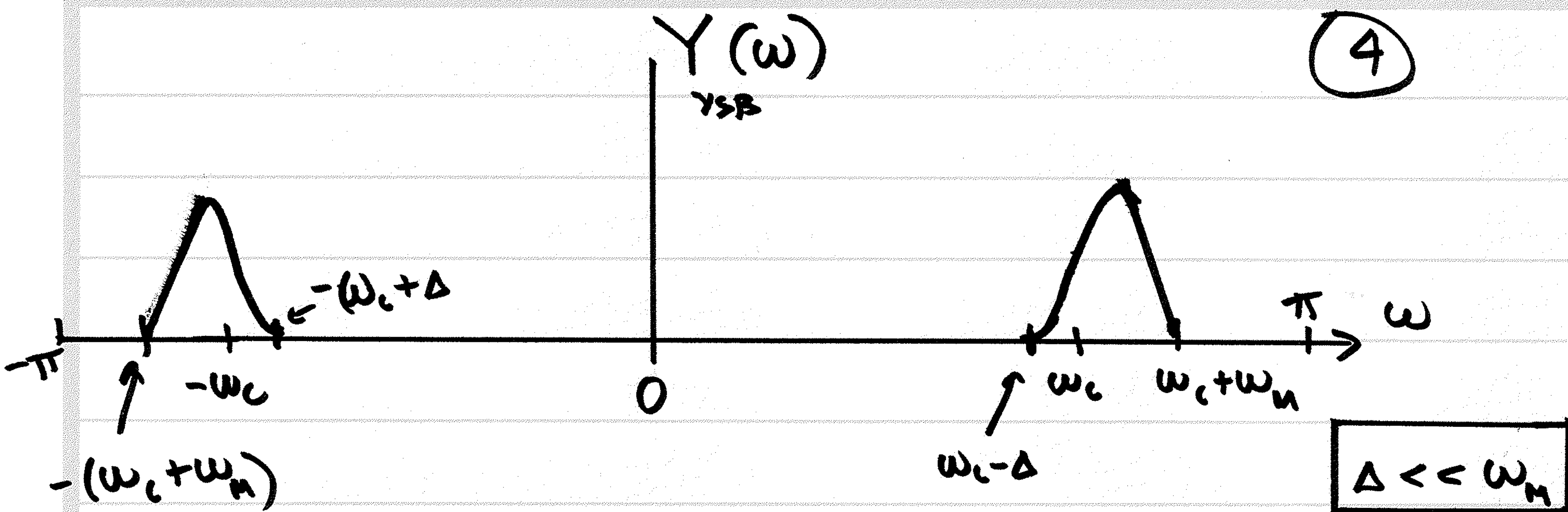
$$y_{VSB}[n] = \mathrm{Re}\left\{ x_{VSB}[n]\, e^{j\omega_c n} \right\}$$

$$= \mathrm{Re}\left\{ \left(x[n] + j\,\hat{x}[n] \right)\left(\cos(\omega_c n) + j \sin(\omega_c n) \right) \right\}$$

$$= x[n]\cos(\omega_c n) - \hat{x}[n]\sin(\omega_c n)$$

If:

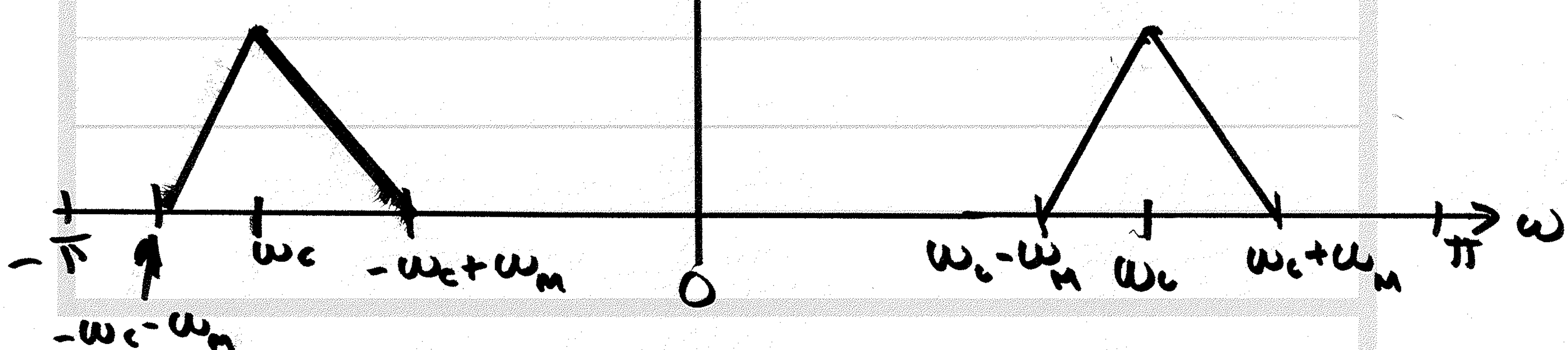




$\Rightarrow$ upper sideband plus trace or vestige of lower sideband (which were the negative frequencies)

• as opposed to:

$$Y_{DSB}(\omega) = \mathrm{DTFT}\left\{x[n]\cos(\omega_c n)\right\}$$



(5)

- Define: $z[n] = x_{VSB}[n]\, e^{j\omega_c n}$

- $y_{VSB}[n] = \mathrm{Re}\{z[n]\} = \frac{1}{2}\{z[n] + z^*[n]\}$

- Note:

IF $x[n] \overset{DTFT}{\longleftrightarrow} X(\omega)$ Then $x^*[n] \overset{DTFT}{\longleftrightarrow} X^*(-\omega)$

- Thus: $Y_{VSB}(\omega) = \frac{1}{2} Z(\omega) + \underbrace{\frac{1}{2} Z^*(-\omega)}_{\text{creates negative frequency portion of spectrum}}$

- again: $\Delta = 0$ is just special case of single sideband modulation

(6)

- Typical mode of operation: create VSB signal in DT domain ⇒

$$x[n] \longrightarrow \boxed{h_{VSB}[n]} \longrightarrow x[n] + j\hat{x}[n]$$

$$x[n] \longrightarrow \boxed{\text{DAC}} \longrightarrow x(t)$$

$$\hat{x}[n] \longrightarrow \boxed{\text{DAC}} \longrightarrow \hat{x}(t)$$

- Transmit:

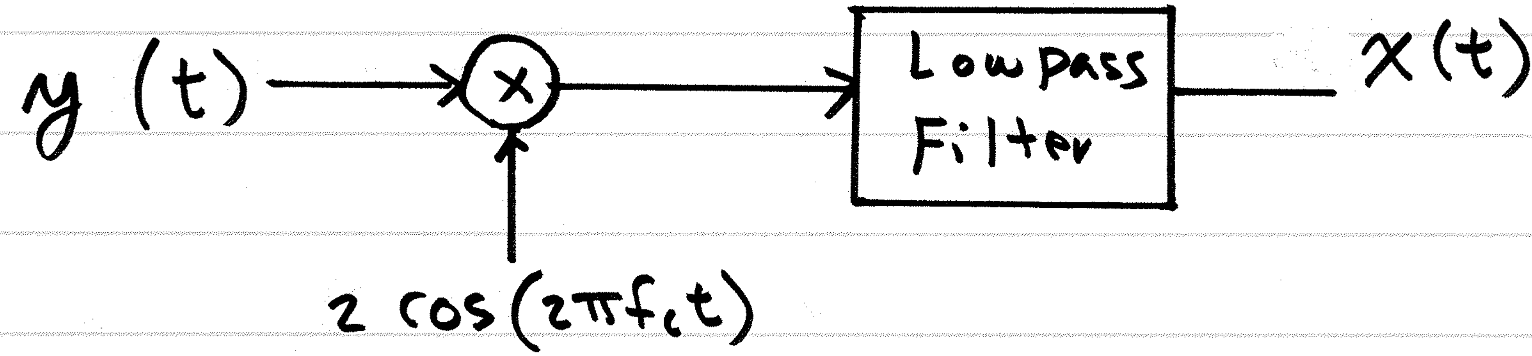
$$y(t) = x(t)\cos(2\pi f_c t) - \hat{x}(t)\sin(2\pi f_c t)$$

- Recall trig identities:

$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$

$$\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

• We recover $x(t)$ via:



Lowpass filter rejects both:

$$x(t)\cos(2\pi(2f_c)t) \quad \text{and}$$

$$\hat{x}(t)\sin(2\pi(2f_c)t)$$

Radio Frequency

• THUS, sending $-\hat{x}(t)\sin(2\pi(2f_c)t)$ along with $x(t)\cos(2\pi f_c t)$ reduces the RF bandwidth $\Rightarrow$ if $\Delta = 0$ $\Rightarrow$ RF bandwidth is halved

VSB modulation

- In Exam 2, Fall 2007, we showed that if

$$h_{VSB}[n] = \begin{cases} 0, & -\pi < \omega < -\Delta \\ H_{VSB}(\omega), & -\Delta < \omega < \Delta \\ 2, & \Delta < \omega < \omega_{max} \end{cases} \qquad (\omega_{max} < \pi)$$

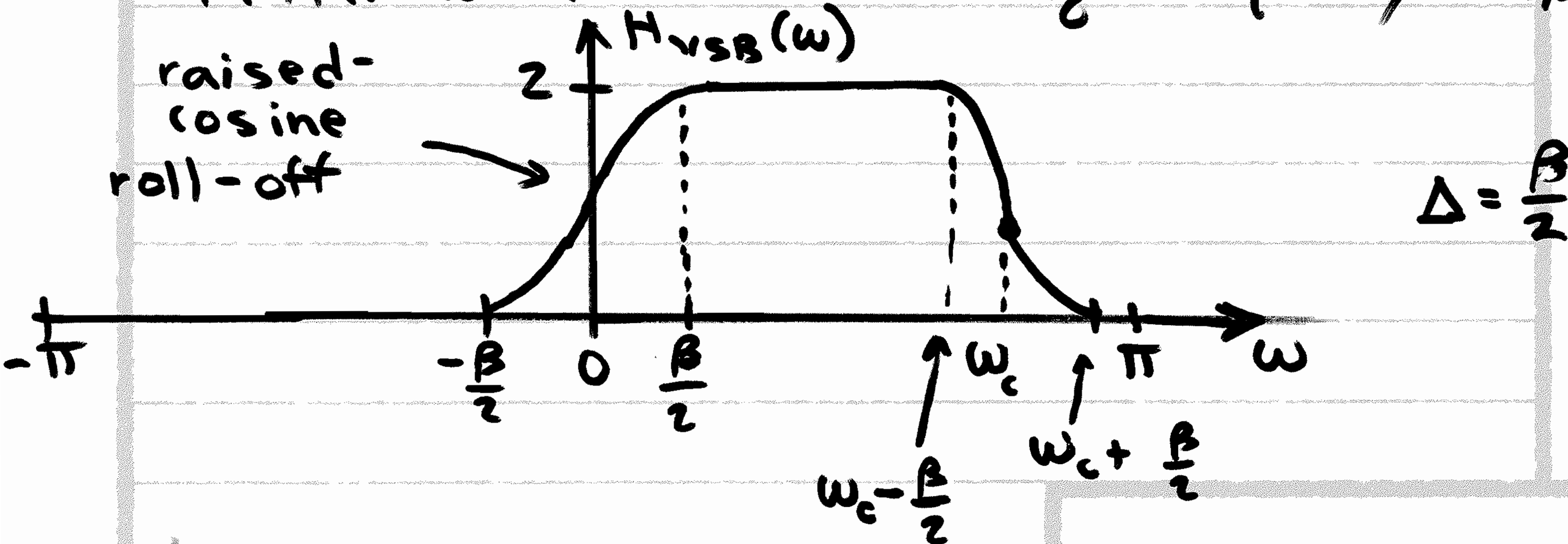
and $H_{VSB}(-\omega) + H_{VSB}(\omega) = 2$ for $|\omega| < \Delta$

then for $y[n] = x[n] * h_{VSB}[n]$

$$y_R[n] = \mathrm{Re}\{y[n]\} = \frac{1}{2}\{y[n] + y^*[n]\}$$

$$= x[n]$$

• For the 8 VSB Digital TV Standard (US) (coming "on-line" in 2009), they use a Filter with the following frequency response



$$H_{VSB}(\omega) = 1 + \cos\left(\frac{\pi}{\beta}\left(\omega - \frac{\beta}{2}\right)\right), \quad -\frac{\beta}{2} < \omega < \frac{\beta}{2}$$

Check if condition is satisfied

③

$$H_{VSB}(-\omega) = 1 + \cos\left(\frac{\pi}{\beta}\left[(-\omega) - \frac{\beta}{2}\right]\right)$$

$$= 1 + \cos\left(\frac{\pi}{\beta}\left(\omega + \frac{\beta}{2}\right)\right)$$

$$= 1 + \cos\left(\frac{\pi}{\beta}\omega + \frac{\pi}{2}\right)$$

$$= 1 - \sin\left(\frac{\pi}{\beta}\omega\right)$$

whereas:

$$H_{VSB}(\omega) = 1 + \cos\left(\frac{\pi}{\beta}\left(\omega - \frac{\beta}{2}\right)\right)$$

$$= 1 + \cos\left(\frac{\pi}{\beta}\omega - \frac{\pi}{2}\right)$$

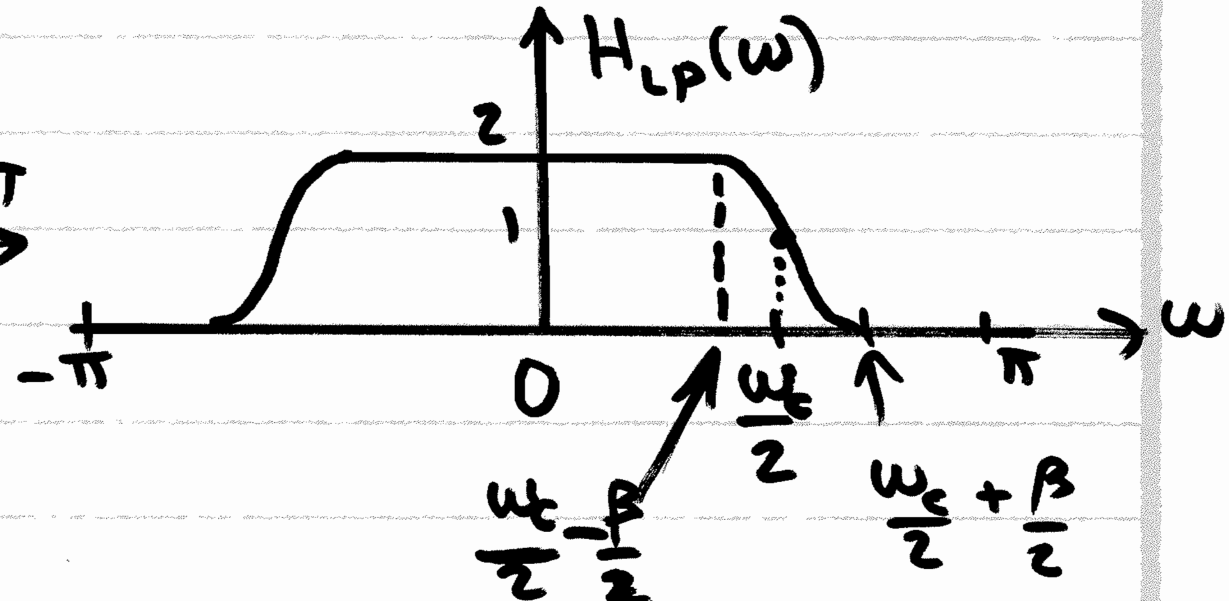
$$= 1 + \sin\left(\frac{\pi}{\beta}\omega\right)$$

Hence: $H_{VSB}(\omega) + H_{VSB}(-\omega) = 2$ ✓

• Note: $h_{VSB}[n] = e^{j\frac{\omega_c}{2}n}\, h_{LP}[n]$

where:

$\underbrace{h_{LP}[n]}_{\text{real-valued and even-symmetric}} \overset{DTFT}{\longleftrightarrow}$



$$H_{LP}(\omega) = \begin{cases} 2, & 0 < |\omega| < \frac{\omega_c}{2} - \frac{\beta}{2} \\ 1 + \cos\left(\frac{\pi}{\beta}\left(|\omega| - \left[\frac{\omega_c}{2} - \frac{\beta}{2}\right]\right)\right), & \frac{\omega_c}{2} - \frac{\beta}{2} < |\omega| < \frac{\omega_c}{2} + \frac{\beta}{2} \\ 0, & \frac{\omega_c}{2} + \frac{\beta}{2} < |\omega| < \pi \end{cases}$$