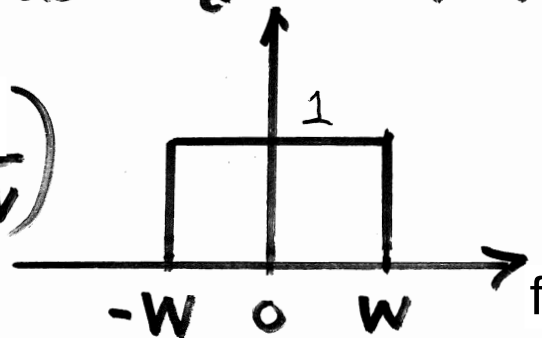


# Sine Functions

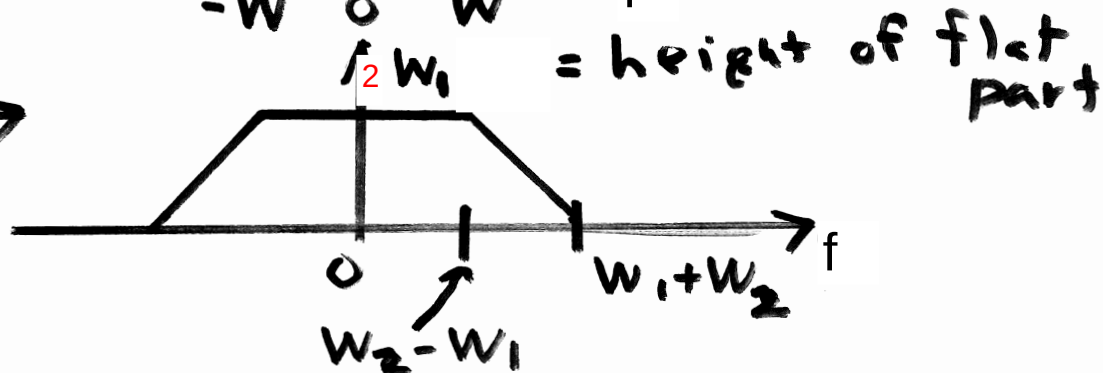
Used as signals OR impulse responders on Exams

$$\frac{\sin^{(2\pi)} Wt}{\pi t} \longleftrightarrow \text{rect}\left(\frac{f}{2W}\right)$$



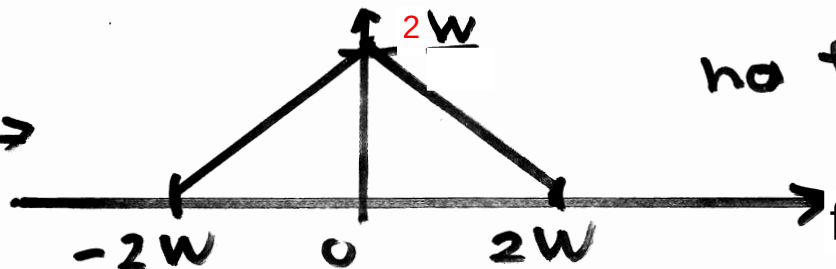
$$\frac{\sin^{(2\pi)} W_1 t}{\pi t} \frac{\sin^{(2\pi)} W_2 t}{\pi t} \longleftrightarrow$$

$$W_2 \geq W_1$$



Special Case:  $W_2 = W_1$ :

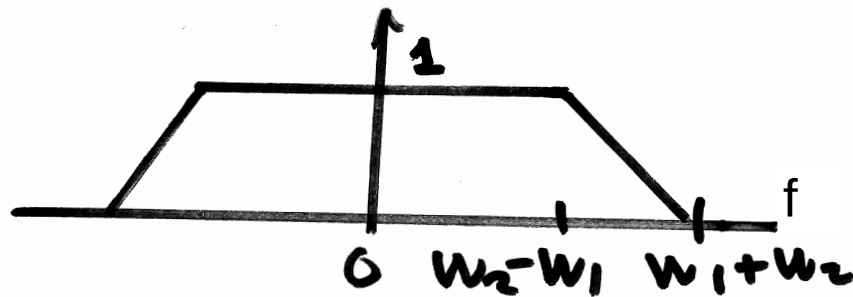
$$\left\{ \frac{\sin^{(2\pi)} Wt}{\pi t} \right\}^2 \longleftrightarrow$$



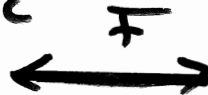
no flat part

Simple Linearity implies

$$\frac{1}{W_1} \frac{\sin^{(2\pi)} W_1 t}{\pi t} \frac{\sin^{(2\pi)} W_2 t}{\pi t} \longleftrightarrow$$



Real and even-symmetric  
in time-domain



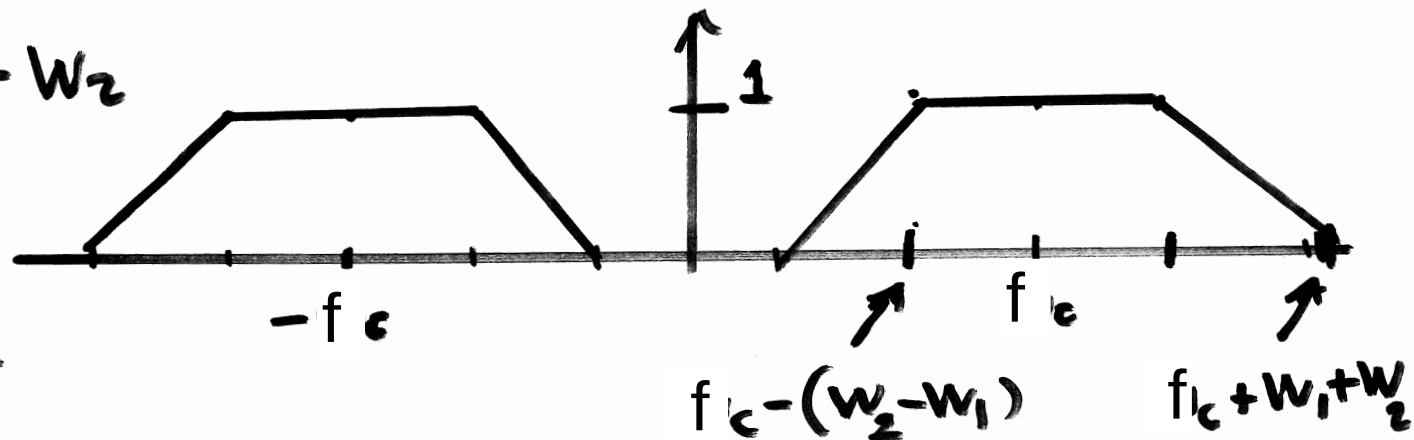
Real and even-symmetric  
in frequency domain

$$\frac{1}{2} \frac{\sin(2\pi W_1 t)}{\pi t} \frac{\sin(2\pi W_2 t)}{\pi t} \xrightarrow{f_c} 2 \cos(2\pi f_c t)$$

$$W_2 \geq W_1$$

$$f_c \geq W_1 + W_2$$

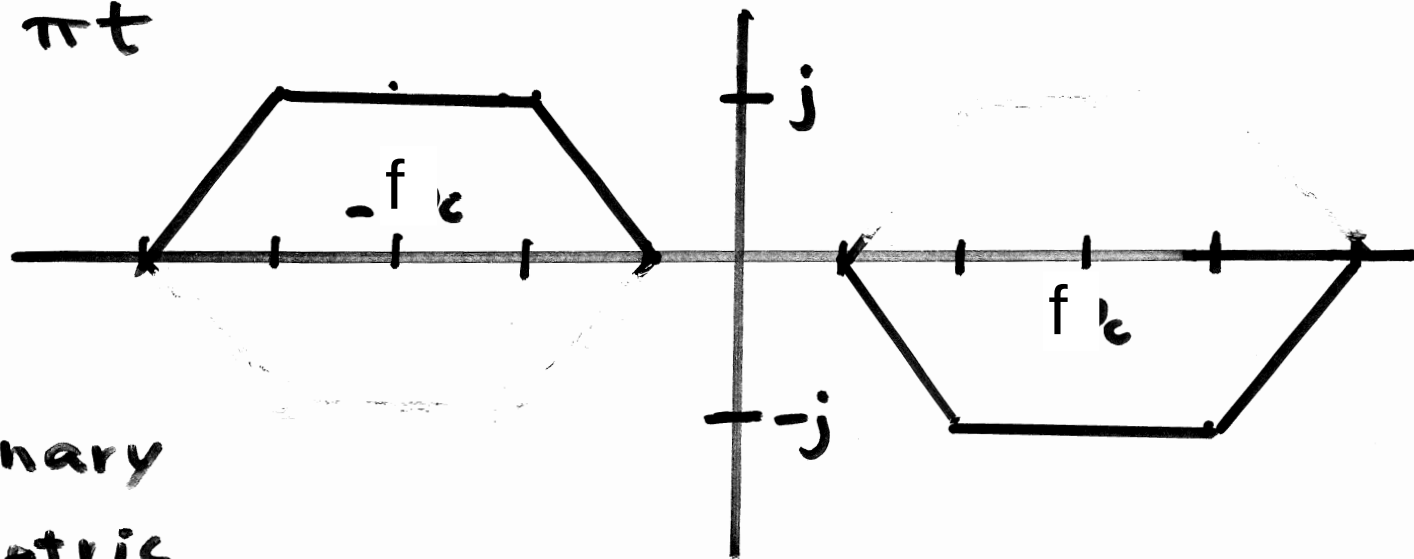
Still real-valued  
and even-symmetric



$$\frac{1}{2} \frac{\sin(2\pi W_1 t)}{\pi t} \frac{\sin(2\pi W_2 t)}{\pi t} \xrightarrow{f_c} 2 \sin(2\pi f_c t)$$

$$f_c \geq W_1 + W_2$$

Now,  
purely imaginary  
and odd-symmetric

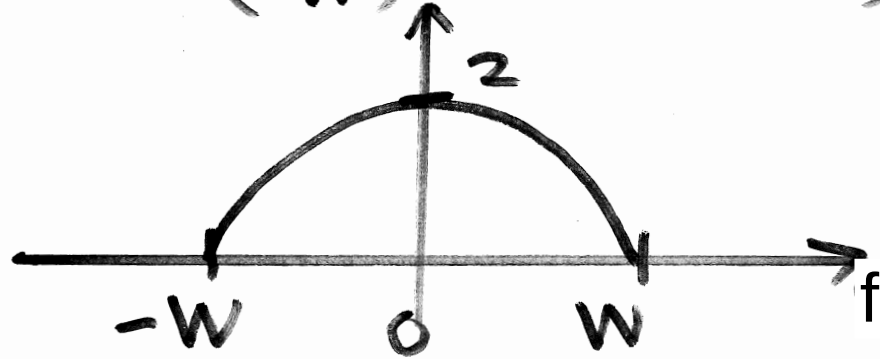


$$\frac{\sin^{(2\pi)} W(t-t_0)}{\pi(t-t_0)} + \frac{\sin^{(2\pi)} W(t+t_0)}{\pi(t+t_0)}$$

$$\xleftrightarrow{+} \text{rect}\left(\frac{f}{2W}\right) \left\{ e^{-j 2\pi f t_0} + e^{j 2\pi f t_0} \right\}$$

$$= \text{rect}\left(\frac{f}{2W}\right) 2 \cos(2\pi f t_0)$$

If  $t_0 = \frac{1}{4W} \Rightarrow \text{rect}\left(\frac{f}{2W}\right) 2 \cos\left(\frac{\pi f}{2W}\right)$



real-valued  
and

even-symmetric

$$\frac{\sin^{(2\pi)}(W(t-t_0))}{\pi(t-t_0)} - \frac{\sin^{(2\pi)}(W(t+t_0))}{\pi(t+t_0)}$$

$$\xleftrightarrow{\mathcal{F}} \text{rect}\left(\frac{f}{2W}\right) \{e^{-j2\pi f t_0} - e^{j2\pi f t_0}\} \frac{2j}{2j}$$

$$= \text{rect}\left(\frac{f}{2W}\right) 2j \sin^{(2\pi)}(f t_0) (-1)$$

$$\text{If } t_0 = \frac{1}{2W} \Rightarrow -\text{rect}\left(\frac{f}{2W}\right) 2j \sin\left(\frac{\pi f}{W}\right)$$

purely imaginary  
and  
odd-symmetric

