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Sampling Theory

See Fig. 6.1.1 on page 388

- $x_s(t) = x_a(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$$\xleftrightarrow{\mathcal{F}} X_a(f) * \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - kF_s) = X_s(f) \qquad F_s = \frac{1}{T_s}$$

$$X_s(f) = F_s \sum_{k=-\infty}^{\infty} X_a(f - kF_s)$$

- Sampling in time $\longrightarrow$ periodic replication in frequency
- This result obtained by using
  (a) property of Fourier Transform
  multiplication in time $\Rightarrow$ convolution in frequency

and

(b) Fourier Transform Pair

$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{T_s}\sum_{k=-\infty}^{\infty} \delta\left(f-k\frac{1}{T_s}\right)$$

ALTERNATIVELY, compute FT of $x_s(t)$ directly:

$$X_s(f) = \mathcal{F}\{x_s(t)\} = \mathcal{F}\left\{x_a(t)\sum_n \delta(t-nT_s)\right\}$$

$$= \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} x_a(nT_s)\,\delta(t-nT_s)\right\}$$

$$= \sum_{n=-\infty}^{\infty} x_a(nT_s)\, e^{-j2\pi f nT_s}$$

③

Define relationship between DT frequency variable ω and CT frequency f

$$\omega = 2\pi f T_s = 2\pi \frac{f}{F_s}$$

$$x[n] = x_a(nT_s)$$

Then:

$$X_s\left(\frac{\omega F_s}{2\pi}\right) = \sum_n x[n] e^{-j\omega n} = X(\omega)$$

$\Rightarrow$ This the DTFT !

So, substitute f by $\frac{\omega F_s}{2\pi}$ in our previous expression for $X_s(f)$:

$$X(\omega) = X_s\left(\frac{\omega F_s}{2\pi}\right) = F_s \sum_{k=-\infty}^{\infty} X_a\left(\frac{\omega F_s}{2\pi} - k F_s\right)$$

$$= F_s \sum_k X_a\left(\frac{F_s}{2\pi}(\omega - k2\pi)\right)$$