

For  $N=21$  and  $N=101$ ,

①

$$D = (N-1)/2$$

$$a = 0.8$$

$$x[n] = a^{|n-D|} \xleftrightarrow{\text{DTFT}} X(\omega) = \frac{1-a^2}{1-2a\cos\omega+a^2} e^{-jD\omega}$$

Plot:

$$X_r(\omega) = \sum_{k=0}^{N-1} X_N(k) P(\omega - k \frac{2\pi}{N})$$

$$P(\omega) = \frac{\sin(\frac{N}{2}\omega)}{N \sin(\frac{1}{2}\omega)} e^{-j\frac{(N-1)}{2}\omega}$$

$$X_N(k) = X(\omega) \Big|_{\omega = \frac{2\pi k}{N}}, \quad k=0, 1, \dots, N-1$$

Plot  $|X(\omega)|$  and  $|X_r(\omega)|$  on same graph

$$x_T[n] = a^{|n-D|}, \quad n=0, 1, \dots, N-1$$

(2)

$$\hat{x}[n] \xleftrightarrow[N]{\text{DFT}} X_N(k) = X(\omega)$$

$$\omega = k \frac{2\pi}{N}$$

$$k=0, 1, \dots, N-1$$

$$x_a[n] = x[n+N] + x[n] + x[n-N]$$

$$\text{for } n=0, 1, \dots, N-1$$

$$\text{and } x[n] = a^{|n-D|}$$

Plot  $x_T[n]$ ,  $\hat{x}[n]$ , and  $x_a[n]$   
all on same graph