EE 538 DSP I Exam 1 Sol'n. F'99

Sol'n. to Prob. 1

Sol'n.: cross-correlate with a user's code. Since code length is 2, take every other value and divide by 2.

$$r_{xc_1}[\ell] = \{\underset{\uparrow}{0}, -2, -4, -2, 2, 2, 0, 2, \overset{2}{\vee}\}$$

$$b_1[n] = \frac{1}{2} r_{xc_1}[2n+1]$$

$$= \frac{1}{2}\{-2, -2, 2, 2\} =$$

$$= \{\underset{\uparrow}{-1}, -1, 1, 1\} \Leftarrow \text{ANS.}$$

$$r_{xc_2}[\ell] = \{0, 2, 0, -2, -2, 2, 0, -2, 2\}$$

$$b_2[n] = \frac{1}{2} r_{xc_2}[2n+1]$$

$$= \frac{1}{2}\{2, -2, 2, -2\}$$

$$= \{\underset{\uparrow}{1}, -1, 1, -1\} \Leftarrow \text{ANS.}$$

Sol'n. to Prob. 2

Ⓐ $y[n] = r\sin\theta \; y_1[n-1] + r\cos\theta \; y[n-1]$

Ⓑ $y_1[n] = x[n] - r\sin\theta \; y[n-1] + r\cos\theta \; y_1[n-1]$

- ZT of B:

$$Y_1(z)\left\{1 - z^{-1} r\cos\theta\right\} = X(z) - r\sin\theta \; z^{-1} \; Y(z)$$

Ⓑ
$$Y_1(z) = \underbrace{\left\{\frac{1}{1 - r\cos\theta \, z^{-1}} X(z) - \frac{r\sin\theta \; z^{-1}}{1 - r\cos\theta \; z^{-1}} Y(z)\right\}}$$

- ZT of A:

Ⓐ
$$Y(z) = r\sin\theta \; z^{-1} \; Y_1(z) + r\cos\theta \; z^{-1} \, Y(z)$$

$$= \frac{r\sin\theta \; z^{-1}}{1 - r\cos\theta \; z^{-1}} X(z) - \frac{r^2\sin^2\theta \; z^{-2}}{1 - r\cos\theta \, z^{-1}} Y(z)$$

$$+ \; r\cos\theta \; z^{-1} \; Y(z)$$

$$Y\{z\}\left\{\frac{1 - r\cos\theta \; z^{-1} + r^2 \sin^2\theta \; z^{-2} - r\cos\theta \, z^{-1} \overset{+r^2\cos^2\theta \, z^{-2}}{\wedge}}{1 - r\cos\theta \; z^{-1}}\right\}$$

$$= \frac{r\sin\theta \; z^{-1}}{1 - r\cos\theta \, z^{-1}} X(z)$$

Sol'n. to Prob. 2 (cont.)

Ans to (a) $H(z) = \frac{Y(z)}{X(z)} = \frac{r \sin\theta \, z^{-1}}{1 - 2r\cos\theta \, z^{-1} + r^2 z^{-2}}$

$\Rightarrow = \frac{r \sin\theta \, z}{z^2 - 2r\cos\theta \, z + r^2}$

$H(z) = \frac{r \sin\theta \, z}{(z - re^{j\theta})(z - re^{-j\theta})}$

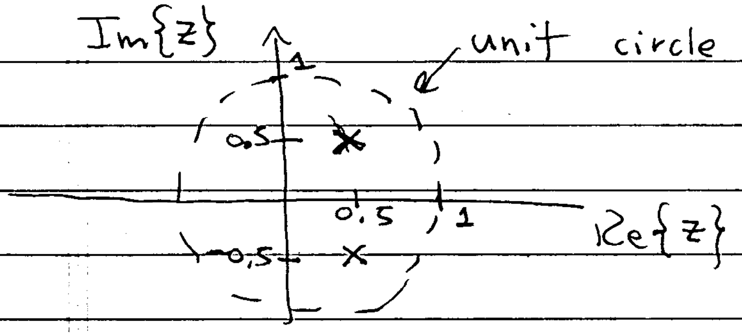
(b) poles are $re^{j\theta}$ and $re^{-j\theta}$

BIBO stability requires $r < 1$ $(|r| < 1)$

(c) when $r = \frac{1}{\sqrt{2}}$ and $\theta = 45^0$, poles are

$P_1 = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}} = \frac{1}{2} + \frac{1}{2} j$

$P_2 = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} = \frac{1}{2} - \frac{1}{2} j$



ROC: $|z| > \frac{1}{\sqrt{2}}$

since causal

Sol'n. to Prob. (2) (cont.)

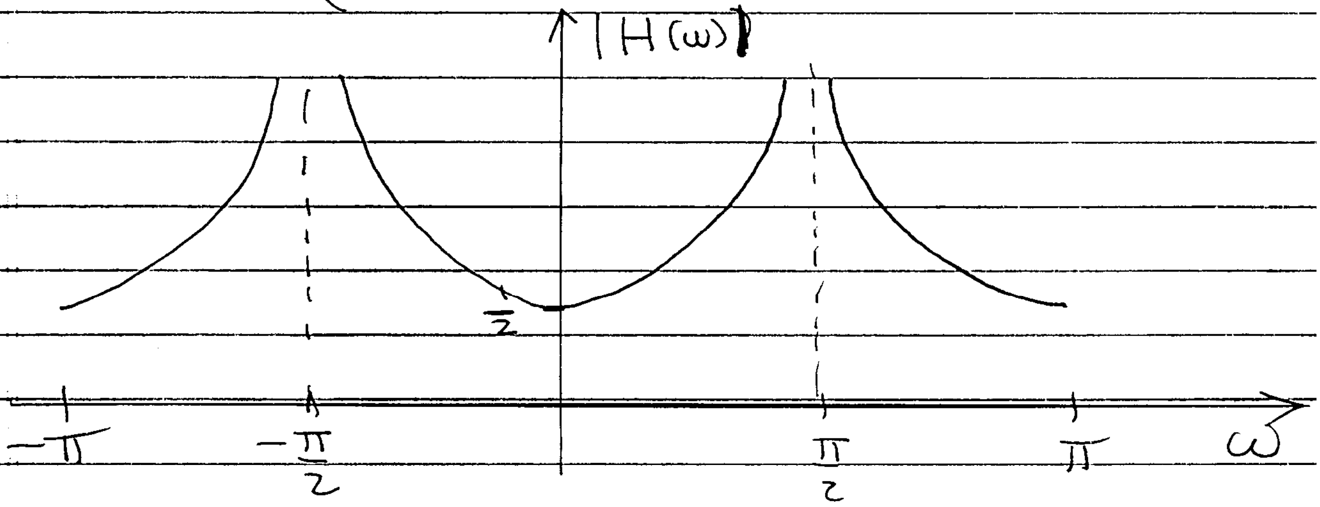
(d) with $r = 1$ and $\theta = 90^\circ$, the poles are

$$p_1 = e^{j\pi/2} = j \quad \text{and} \quad p_2 = e^{-j\pi/2} = -j$$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = \frac{z}{(z-j)(z+j)}\Bigg|_{z=e^{j\omega}}$$

$$= \frac{e^{j\omega}}{(e^{j\omega}+j)(e^{j\omega}-j)}$$

(i)



(ii) $x[n] = e^{j\frac{\pi}{2}n}$ or $x[n] = \cos\left(\frac{\pi}{2}n\right)$

since $\lim_{\omega \to \frac{\pi}{2}} |H(\omega)| = \infty$

Solution to Prob. 3

$$y(t) = x(t) * g(t)$$

$$= x(t) * \{\delta(t) - \delta(t-\tau)\}$$

$$= x(t) - x(t-\tau)$$

$$= \sum_{k=-\infty}^{\infty} b[k] \{p(t-kT_0) - p(t-kT_0-\tau)\}$$

$$y[n] = \sum_{k=-\infty}^{\infty} b[k] \{p(nT_0 - kT_0) - p(\underbrace{nT_0 - kT_0 - \tau}_{(n-k-1)T_0})\}$$

for $\tau = T_0$

$$= \sum_{k=-\infty}^{\infty} b[k] \{p[n-k] - p[n-k-1]\}$$

$$= \sum_{k=-\infty}^{\infty} b[k]\, h[n-k]$$

where: $h[n] = p[n] - p[n-1]$

$$p[n] = p(nT_0) = \tilde{p}[2n] = 4\delta[n]$$

Answer to (a) $\Big\{$ Thus: $h[n] = 4\delta[n] - 4\delta[n-1]$

Sol'n. to Prob. 3 (cont.)

for $\tau = \frac{T_0}{2}$:

$$y[n] = \sum_{k=-\infty}^{\infty} b[k]\left\{p(nT_0 - kT_0) - p\left(nT_0 - kT_0 - \frac{T_0}{2}\right)\right\}$$

$$= \sum_{k=-\infty}^{\infty} b[k]\left\{p\left(2(n-k)\frac{T_0}{2}\right) - p\left([2(n-k)-1]\frac{T_0}{2}\right)\right\}$$

$$= \sum_{k=-\infty}^{\infty} b[k]\, h[n-k]$$

where: $h[n] = \tilde{p}[2n] - \tilde{p}[2n-1]$

$$= 4\delta[n] - \{1, \underset{\underset{n=0}{\uparrow}}{-2}, -2, 1\}$$

$$= \{-1, \underset{\underset{n=0}{\uparrow}}{6}, 2, -1\}$$