Perfect Reconstruction Filter Banks (PRFB)

- Text Sect 11.11

- From one viewpoint, a PRFB is a transmultiplexer with the left and right sides reversed, now called synthesis and analysis sections.
  - That changes everything since the analysis and synthesis sections (which are in series) do not commute.
    - Zero-inserters and decimators are not LTI.
      - They are linear but no TI.
    - Maximal decimation leads to aliasing.
      - Decimating by a factor equal to the number of subbands the signal is decomposed into...
If $g_m[n] = h_M[n] = e^{jm \frac{2\pi}{M} n} h_0[n]$, $m = 0, 1, ..., M-1$

with $h_0[n]$ = LPF passing $(-\pi/M, \pi/M)$ then

PRFB is a transmultiplexer with 2 "sides" reversed.
Efficient Implementation

\[ x[n] \rightarrow h_0[n] \rightarrow M\text{-pt DFT per } n \rightarrow \chi_0[n] \rightarrow h_0[n] \rightarrow \chi_1[n] \rightarrow \text{IDFT per } n \rightarrow h_1[n] \rightarrow \text{Interleaver} \rightarrow y[n] \]

\[ \chi[n] \rightarrow h_m[n] = h_{LP}[Mn-m] \quad m = 0, 2, \ldots, M-1 \]

\[ h^{+}_m[n] = h_{LP}[Mn+m] \]

\[ h_0[n] = h_{LP}[n] = \text{LPF passing } (-\pi/M, \pi/M) \]

"deinterleaver"
• In this course, we will only consider subbands of equal width.

• Dyadic case of subbands of unequal widths leads to wavelets.
  • Unequal widths take advantage of the logarithmic response of our ears as a function of frequency.
The primary application of PRFBs is subband coding for audio compression. Denoising is another big application.

The no. of bits per sample at the output of each subband varies according to the energy in that subband and the response of our ears over that frequency band.

Ultimately, the total no. of bits required to reconstruct the audio signal with "decent" fidelity is reduced \( \Rightarrow \) compression.

But to reduce the no. of bits, must decimate each subband output by a factor equal to the no. of subbands (maximal decimation).

leads to aliasing effects w/ real-world filters