Efficient Implementation for Ideal Case

M uniform subbands

$$x_\ell[n] = \sum_{k=-\infty}^{\infty} x[k]\, h_\ell[n-k] \Big|_{n=Mn}$$

Maximal Decimation

$$h_\ell[n] = e^{j 2\pi \frac{\ell}{M} n} \; M \frac{\sin\left(\frac{\pi}{M} n\right)}{\pi n}$$

$\ell = 0, 1, \dots, M$

$$x_\ell[n] = \sum_{m=0}^{M-1} \sum_{k'=-\infty}^{\infty} x[k'M+m]\, h_\ell[Mn-(k'M+m)]$$

$$= \sum_{m=0}^{M-1} \sum_{k'=-\infty}^{\infty} x[k'M+m]\, h_\ell[M(n-k') - m]$$

minus sign

$$= \sum_{m=0}^{M-1} x[k'M+m] * h_\ell[Mn-m]$$

where: $h_\ell[Mn-m] = e^{j 2\pi \frac{\ell}{M}(Mn-m)} \dfrac{\sin\left(\frac{\pi}{M}(Mn-m)\right)}{\frac{\pi}{M}(Mn-m)}$

$$= \underbrace{e^{-j 2\pi \frac{\ell m}{M}}}_{\text{elements of } M \times M \text{ DFT matrix}} \frac{\sin\left(\pi\left(n-\frac{m}{N}\right)\right)}{\pi(n-m/N)}$$