OFDM Example

Given: $N = 4 \quad L = 3$

$$x[n] = \sum_{k=0}^{3} b_k \, s_k[n]$$

$$h[n] = \{\underset{\uparrow}{j}, 1, -j\}$$

$$= j\delta[n] + \delta[n-1] - j\delta[n-2]$$

- After cyclic prefix is added AND after convolution with channel $h[n]$,

$$\tilde{y}[n] = \left\{\tfrac{1}{2}j,\ \tfrac{1}{2} - \tfrac{1}{2}j,\ -\tfrac{1}{2},\ \underset{\uparrow}{\tfrac{1}{2} + j},\ \tfrac{1}{2},\ \tfrac{1}{2} - j,\ -\tfrac{1}{2},\ \tfrac{1}{2} + \tfrac{1}{2}j,\ \tfrac{-j}{2}\right\}$$

$$n = \underbrace{-3 \quad -2 \quad -1}_{\text{discard } L=3 \text{ pts. at beginning}} \quad \underbrace{0 \quad 1 \quad 2 \quad 3}_{\text{Keep}} \quad \underbrace{4 \quad 5}_{\text{discard } L-1=2 \text{ pts. at end}}$$

$$y[n] = \left\{ \underset{\uparrow}{\frac{1}{2}+j}, \frac{1}{2}, \frac{1}{2}-j, -\frac{1}{2} \right\}$$

Before we proceed, we are going to need

$$H_N(k) = H(\omega)\Big|_{\omega = k\frac{2\pi}{4}} \qquad k = 0, 1, 2, 3$$

$$h[n] = \left\{ \underset{\uparrow}{j}, 1, -j \right\}$$

$$H(z) = j + z^{-1} - j z^{-2}$$

$$H(\omega) = H(z)\Big|_{z = e^{j\omega}}$$

$$H_N(k) = H(z)\Big|_{z = e^{jk\frac{2\pi}{4}}} \quad ; k = 0, 1, 2, 3$$

$$\omega = 0 \Rightarrow z = e^{j0} = 1 \qquad \boxed{k=0}$$

$$\omega = \frac{2\pi}{4} = \frac{\pi}{2} \Rightarrow z = e^{j\frac{\pi}{2}} = j \qquad \boxed{k=1}$$

$$\omega = \frac{2(2\pi)}{4} = \pi \Rightarrow z = e^{j\pi} = -1 \qquad \boxed{k=2}$$

$$\omega = \frac{3(2\pi)}{4} = \frac{3\pi}{2} - \frac{4\pi}{2} \Rightarrow \frac{-\pi}{2} \Rightarrow z = e^{-j\frac{\pi}{2}} = -j \qquad \boxed{k=3}$$

$$H_N(0) = H(z)\Big|_{z=1} = j + (1)^{-1} - j(1)^{-2} = 1$$

$$H_N(1) = H(z)\Big|_{z=j} = j + (j)^{-1} - j(j)^{-2} = j$$

$$H_N(2) = H(z)\Big|_{z=-1} = j + (-1)^{-1} - j(-1)^{-2} = -1$$

$$H_N(3) = H(z)\Big|_{z=-j} = j + (-j)^{-1} - j(-j)^{-2} = 3j$$

Also, we need the four sinewaves:

$$s_0[n] = e^{j0\frac{2\pi}{4}n}\{u[n]-u[n-4]\} = \{1,1,1,1\}$$

$$s_1(n) = e^{j\frac{2\pi}{4}n}\{u(n)-u(n-4)\} = \{1,j,-1,-j\}$$

$$s_2[n] = e^{j2(\frac{2\pi}{4})n}\{u[n]-u[n-4]\} = \{1,-1,1,-1\}$$

$$s_3[n] = e^{j3(\frac{2\pi}{4})n}\{u[n]-u[n-4]\} = \{1,-j,-1,j\}$$

Need to compute:

$$b_k = \frac{1}{4}\sum_{n=0}^{3} y[n]\, s_k^*(n) \Big/ H_N(k)$$

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$$b_0 = \frac{(1)\left(\frac{1}{2}+j\right) + (1)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}-j\right) + (1)\left(-\frac{1}{2}\right)}{1}$$

$$= 1 \qquad \boxed{b_0 = 1}$$

$$b_1 = \frac{(1)\left(\frac{1}{2}+j\right) + (j)^*\left(\frac{1}{2}\right) + (-1)\left(\frac{1}{2}-j\right) + (-j)^*\left(-\frac{1}{2}\right)}{j}$$

$$= 1 \qquad \boxed{b_1 = 1}$$

Note relative to all 4 values here: the values given in statement of problem were computed via ifft in matlab (Inverse FFT) which already includes division by N=4

$$b_2 = \frac{(1)\left(\frac{1}{2}+j\right) + (-1)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}-j\right) + (-1)\left(-\frac{1}{2}\right)}{(-1)}$$

$$= -1 \qquad \boxed{b_2 = -1}$$

$$b_3 = \frac{(1)\left(\frac{1}{2}+j\right) + (-j)^*\left(\frac{1}{2}\right) + (-1)\left(\frac{1}{2}-j\right) + (j)^*\left(-\frac{1}{2}\right)}{3j}$$

$$= 1 \qquad \boxed{b_3 = 1}$$