Interesting problem on old Exam 1's involving Notch Filters and All-Pass Filters

- Two LTI systems in parallel (single-pole / single-zero)

$$H_1(z) = \frac{z - \frac{1}{p^*}}{z - p} \Rightarrow \text{all-pass} \Rightarrow |H(\omega)| = \frac{1}{|p|} \quad \forall \omega$$

$$H_2(z) = \frac{z + \frac{1}{p^*}}{z + p} \Rightarrow \text{all-pass} \Rightarrow |H(\omega)| = \frac{1}{|p|} \quad \forall \omega$$

- BUT the parallel combination is a Notch Filter!

$H(z) = H_1(z) + H_2(z) \Rightarrow$ place over common denominator

$$= \frac{\left(z - \frac{1}{p^*}\right)(z+p) + (z-p)\left(z + \frac{1}{p^*}\right)}{(z-p)(z+p)}$$

- Examine/Simplify Numerator:

$$z^2 + pz - \frac{1}{p^*}z - \frac{p}{p^*} + z^2 - pz + \frac{1}{p^*}z - \frac{p}{p^*}$$

$$= 2\left(z^2 - \frac{p}{p^*}\right) = 2\left(z - \sqrt{\frac{p}{p^*}}\right)\left(z + \sqrt{\frac{p}{p^*}}\right)$$

- <u>Note:</u> $\frac{p}{p^*}$ is on unit circle $\Rightarrow p = |p| e^{j\angle p}$

$$\sqrt{\frac{p}{p^*}} = e^{j\angle p} \qquad -\sqrt{\frac{p}{p^*}} = e^{j(\pi + \angle p)}$$

- Thus: parallel combination has two zeroes on the unit circle and thus notches out the frequencies $\angle p$ and $\pi + \angle p$ (or $\angle p - \pi$)

$\underbrace{\qquad\qquad\qquad}_{\text{depending on which is in } -\pi < \omega < \pi}$