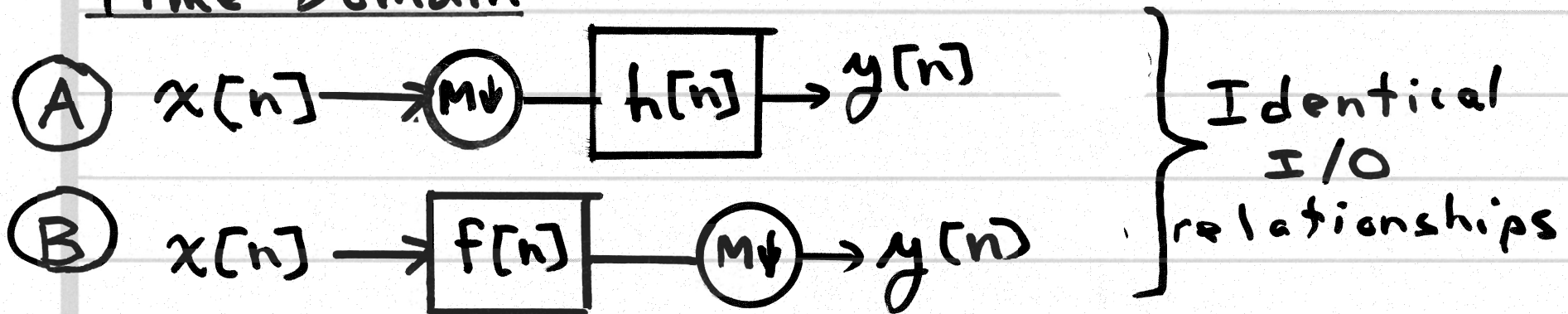


Noble's Identity II

(1)

The two DT systems below have identical I/O relationships

Time Domain:



$$\text{If: } f[n] = \sum_{k=-\infty}^{\infty} h[k] \delta[n - kM] \xleftrightarrow{\text{DTFT}} F(\omega) = H(M\omega)$$

Frequency Domain Proof:

$$\text{System A: } Y(\omega) = H(\omega) \frac{1}{M} \sum_{k=0}^{M-1} X\left(\frac{\omega - k2\pi}{M}\right)$$

$$\text{System B: } Y(\omega) = \frac{1}{M} \sum_{k=0}^{M-1} F\left(\frac{\omega - k2\pi}{M}\right) X\left(\frac{\omega - k2\pi}{M}\right)$$

• If $F(\omega) = H(M\omega)$, then

②

$$F\left(\frac{\omega - k2\pi}{M}\right) = H\left(M\left(\frac{\omega - k2\pi}{M}\right)\right)$$

$$= H(\omega - k2\pi) = H(\omega)$$

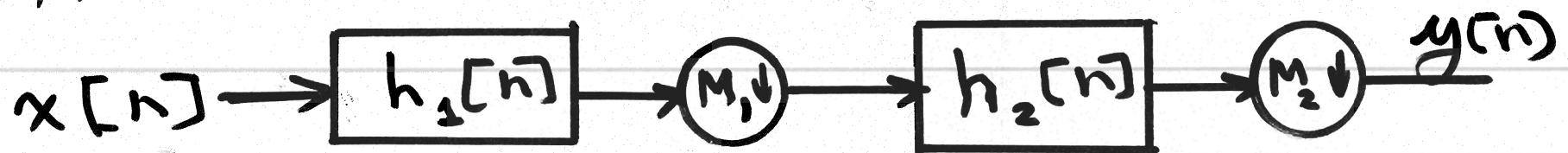
• Since DTFT is periodic with period 2π

• So with $F(\omega) = H(M\omega)$, system B'_S I/O is also:

$$Y(\omega) = H(\omega) \sum_{k=0}^{M-1} X\left(\frac{\omega - k2\pi}{M}\right)$$

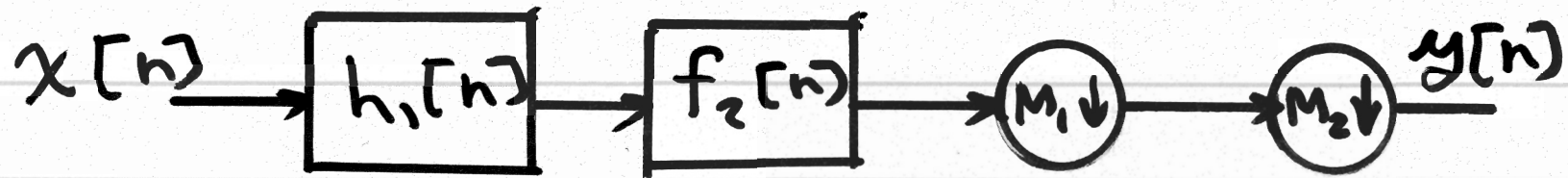
• End of Proof

Application / Example / Typical Use



• Use Noble II to move 1st decimator to other side of 2nd filter

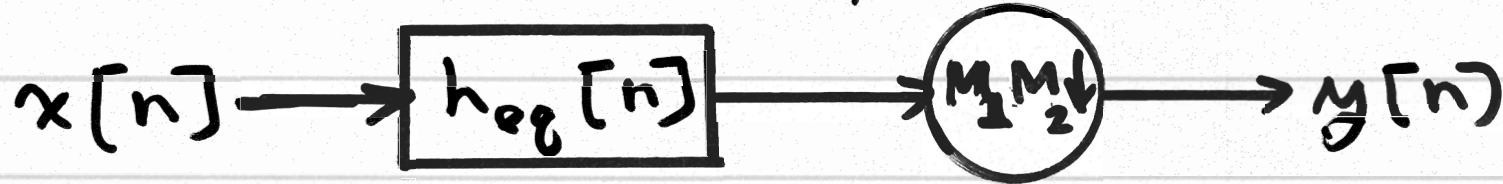
3



$$f_2[n] = \sum_{k=-\infty}^{\infty} h_2[k] \delta[n - kM_1]$$

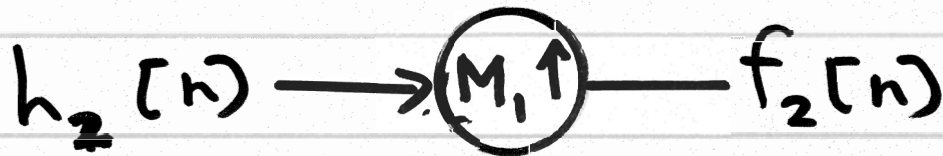
reduces to:

product of decimation factors



where:

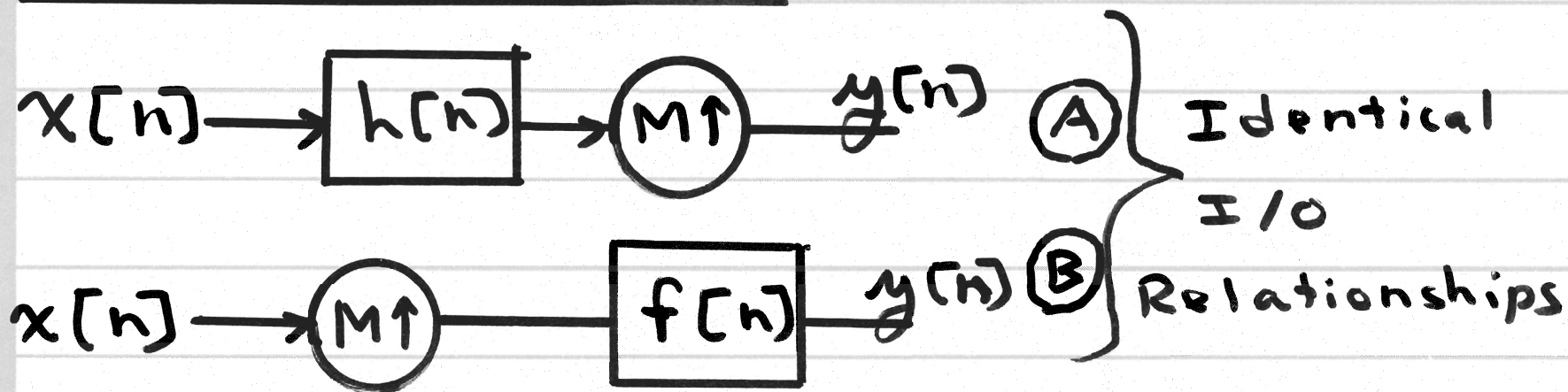
$$h_{eq}[n] = h_1[n] * f_2[n] \xleftrightarrow{\text{DTFT}} H(\omega) = H_1(\omega) H_2(M_1, \omega)$$



insert $M_1 - 1$ zeros between each pair of successive values of $h_2[n]$ to form $f_2[n]$

Noble I Identity

(4)



where:
$$f[n] = \sum_{k=-\infty}^{\infty} h[k] \delta[n-kM]$$

$$h[n] \xrightarrow{(M\uparrow)} f[n] \xleftrightarrow{\text{DTFT}} F(\omega) = H(M\omega)$$

Frequency Domain Proof

System A I/O $\Rightarrow Y(\omega) = H(M\omega) X(M\omega)$

System B I/O $\Rightarrow Y(\omega) = X(M\omega) F(\omega)$

If $F(\omega) = H(M\omega) \Rightarrow$ systems have same I/O