Due Date: Sunday, Dec. 1, by midnight EST. Fall 2024

Note: You should make use of the matlab codes PR4chan.m and PRRC4chan.m posted at the course web site.

Background. See relevant notes at course web site.

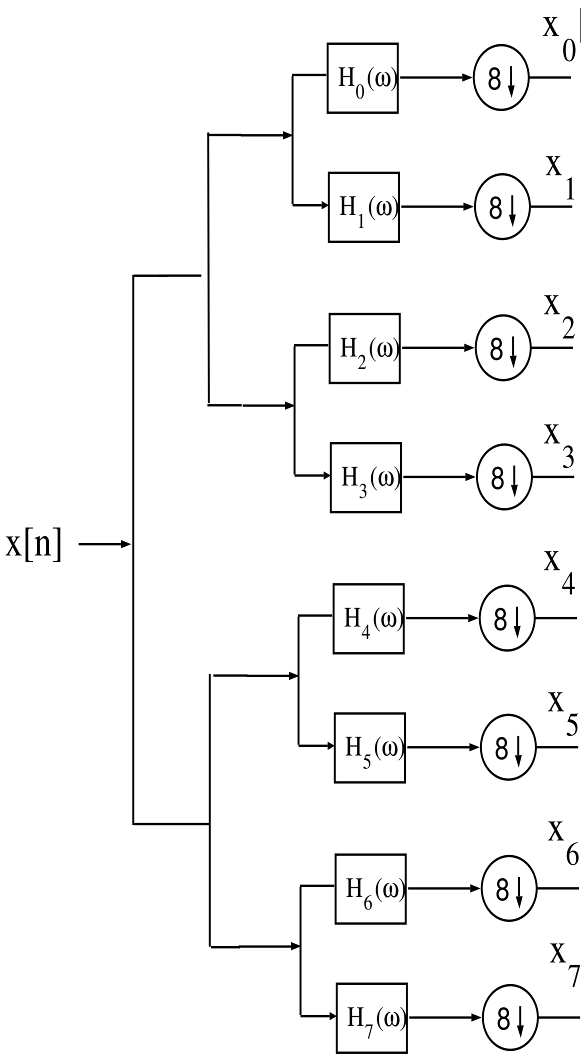

Figure 1(a). Analysis Filter Bank, $M = 8$.

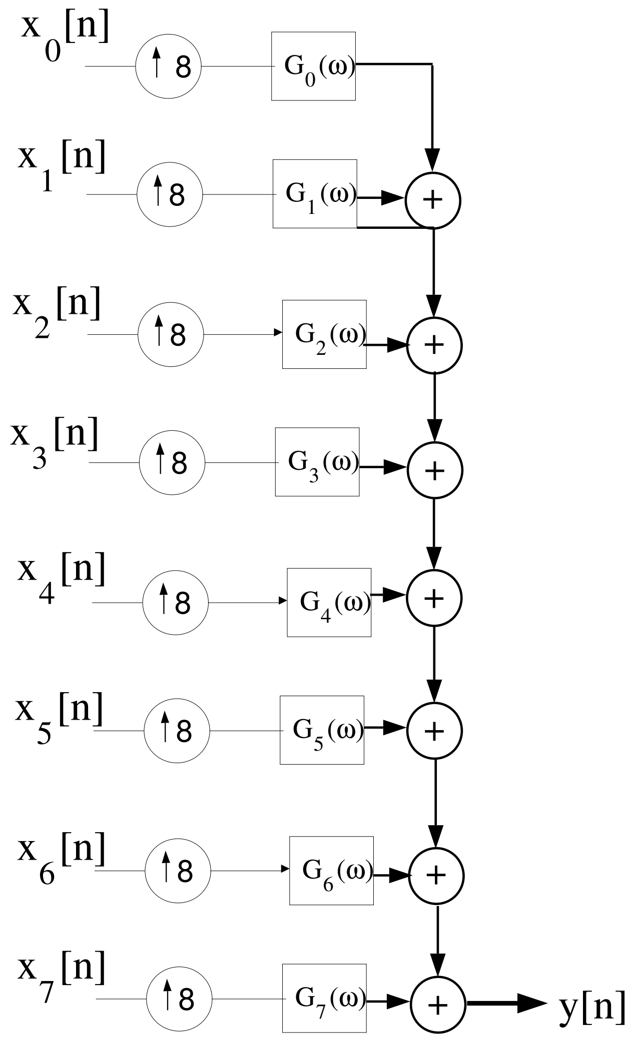

Figure 1(b). Synthesis Filter Bank, $M = 8$.

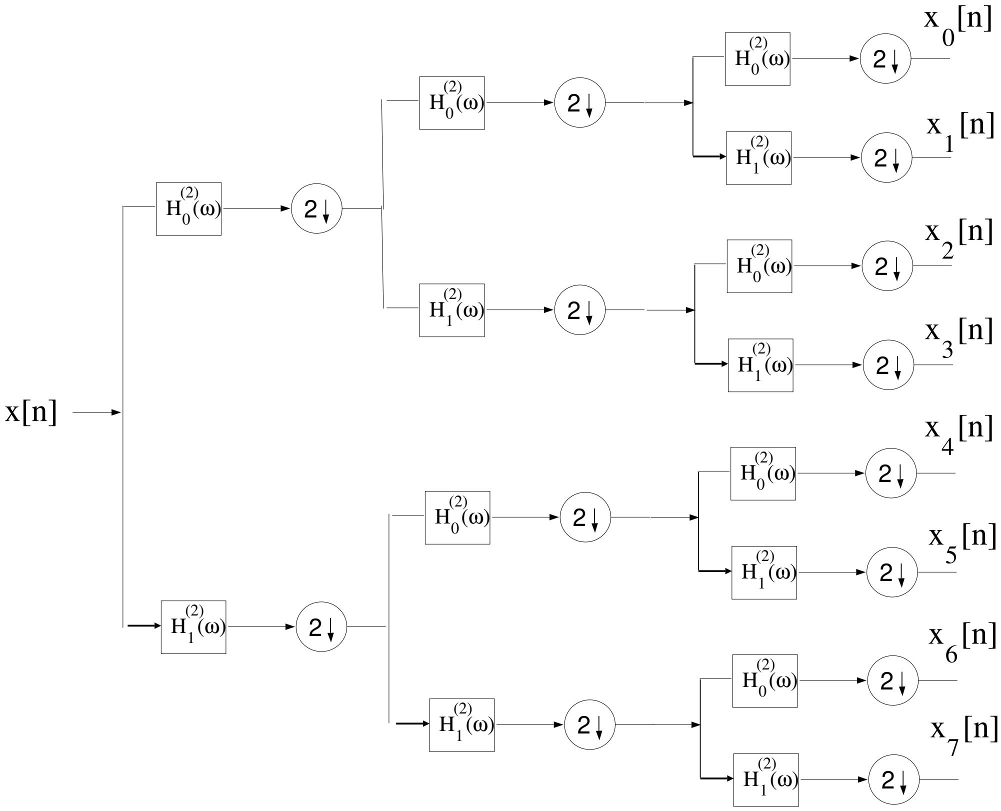

Figure 2(a). Analysis Section of Three-Stage Tree-Structured Filter Bank.

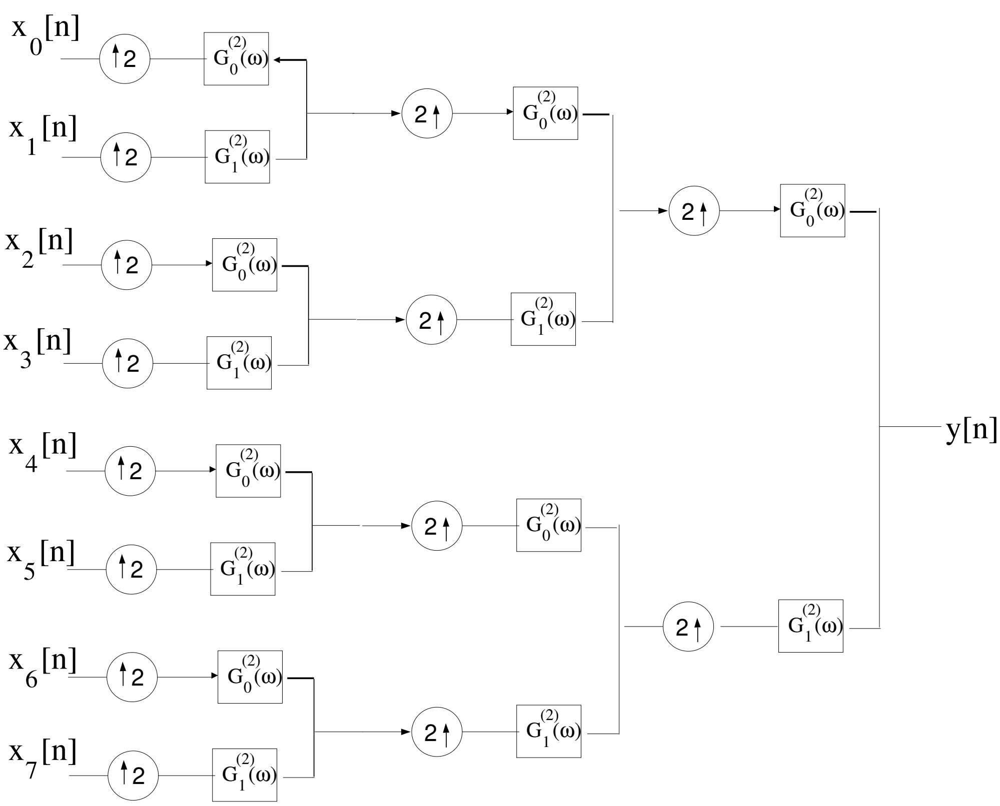

Figure 2(b). Synthesis Section of Three-Stage Tree-Structured Filter Bank.

Synthesizing M=8 Channel Perfect Reconstruction Filter Banks from Tree-Structured Filter Banks. This Matlab assignment is centered on synthesizing an M=8 channel uniform PR filter bank from a three stage tree-structured PR filter bank. As discussed in class, if the number of channels, M, is a power of two, an M-channel uniform PR filter bank may be synthesized via an equivalent (same I/O relationship) tree-structured PR filter bank having $\log_2(M)$ stages with each stage formed from a two-channel QMF filter bank as depicted in Figure 2, i.e., the combination of the analysis filter pair, $\{H_0^{(2)}(\omega), H_1^{(2)}(\omega)\}$, and synthesis filter pair $\{G_0^{(2)}(\omega), G_1^{(2)}(\omega)\}$, form a 2-channel PR filterbank.

Part I. Deriving the Uniform Filter Bank Equivalent to Tree-Structured Filter Bank. Using Noble's Decimation Identity to express each analysis filter, $H_m(\omega)$, $m = 0, 1, ..., 7$, in terms of $H_0^{(2)}(\omega)$ and $H_1^{(2)}(\omega)$. In each case, express the corresponding impulse response $h_m[n]$, $m = 0, 1, ..., 7$, in terms of $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$.

Next, use Noble's Upsampling Identity to express each synthesis filter, $G_m(\omega)$, $m = 0, 1, ..., 7$, in terms of $G_0^{(2)}(\omega)$ and $G_1^{(2)}(\omega)$. In each case, express the corresponding impulse response $g_m[n]$, $m = 0, 1, ..., 7$, in terms of $g_0^{(2)}[n]$ and $g_1^{(2)}[n]$. Note $g_0^{(2)}[n] = h_0^{(2)}[n]$ and $g_1^{(2)}[n] = -h_1^{(2)}[n]$; this is assumed throughout.

Part II. Matlab Calculations. For **each pair** of $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$ specified below, use Matlab and the results derived above to compute the numerical values of the analysis filters $h_m[n]$, $n = 0, 1, ..., N-1$, for $m = 0, 1, ..., 7$. Plot all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$, superimposed on a single graph using a 4096 pt. FFT of each $h_m[n]$, $m = 0, 1, ..., 7$. Next, compute the numerical values of the synthesis filters $g_m[n]$, $n = 0, 1, ..., N-1$, for $m = 0, 1, ..., 7$. Also, using Matlab, for each case below place each impulse response $h_m[n]$, $n = 0, 1, ..., 7$, as the row of a matrix called $\mathbf{H}$ and compute $\mathbf{H}\mathbf{H}^H$. Note superscript H denotes Conjugate-Transpose or Hermitian-Transpose; in Matlab, it's tic-mark. The matrix $\mathbf{H}\mathbf{H}^H$. contains the inner product between every pair of two filters; it will be a scalar multiple of the Identity Matrix if the filters are orthogonal. Put the elements of the resulting 8×8 matrix in a Table. Finally, generate a sample function of Gaussian random process with zero mean and a variance of 1, of length 128 as the input signal $x[n]$. Plot the magnitude of the DTFT of $x[n]$ using a 4096 pt. FFT; also plot the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank $y[n]$, using a 4096 pt. FFT.

(A) $h_0^{(2)}[n] = \frac{1}{\sqrt{2}}\{1, 1\}$ and $h_1^{(2)}[n] = \frac{1}{\sqrt{2}}\{1, -1\}$.

- (i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 1(a).
- (ii) Label the table containing the values of the 8×8 matrix $\mathbf{H}\mathbf{H}^H$ as Table 1.
- (i) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 1(b).
- (iv) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 1(c).

(B) $h_0^{(2)}[n] = h_{sr}[n-16]$, $n = 0, 1, ..., 31$, $h_1^{(2)}[n] = (-1)^n h_0^{(2)}[n]$, and $\beta = 0.35$ where

$$h_{sr}[n] = \sqrt{2}\left\{\frac{2\beta\cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]}\right\}, n = -16, ..., 1, ..., 15.$$

(i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 2(a).

(ii) Label the table containing the values of the 8×8 matrix $\mathbf{HH}^H$ as Table 2.

(iii) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 2(b).

(iv) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 2(c).

(C) $h_0^{(2)}[n] = h_{sr}[n-24]$, $n = 0, 1, ..., 47$, $h_1^{(2)}[n] = (-1)^n h_0^{(2)}[n]$, and $\beta = 0.1$ where

$$h_{sr}[n] = \sqrt{2}\left\{\frac{2\beta\cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]}\right\}, n = -24, ..., 1, ..., 23.$$

(i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, ..., 7$ *superimposed* as Figure 3(a).

(iii) Label the table containing the values of the 8×8 matrix $\mathbf{HH}^H$ as Table 3.

(iv) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 3(b).

(v) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 3(c).

General Information.

Deliverables for this project include:

- The Deliverables for Part I. You can show the work for just one of the 8 "chains" on both the analysis side and the synthesis side. On the analysis side, choose a chain that has at least one instance of $H_0^{(2)}(\omega)$ and one instance of $H_1^{(2)}(\omega)$. On the synthesis side, choose a chain that has at least one instance of $G_0^{(2)}(\omega)$ and one instance of $G_1^{(2)}(\omega)$.
- 9 plots and 3 tables
- a paragraph summarizing your observations and any conclusions you can draw from this set of computer experiments.
- your source code appended to the report

The collection of plots and accompanying explanation should be put together in a cohesive manner in the form of a brief report. You may use any Matlab command you like in solving these problems. Each student is expected to do his/her own work and each must turn in his/her own report. Again, your write-up for this homework should be in the form of a brief report. Handwriting is acceptable but please be sure it is legible.