

Due Date: Tuesday, Nov. 21, by midnight EST. Fall 2023

Note: You should make use of the matlab codes PR4chan.m and PRRC4chan.m posted at the course web site.

Background. See relevant notes at course web site.

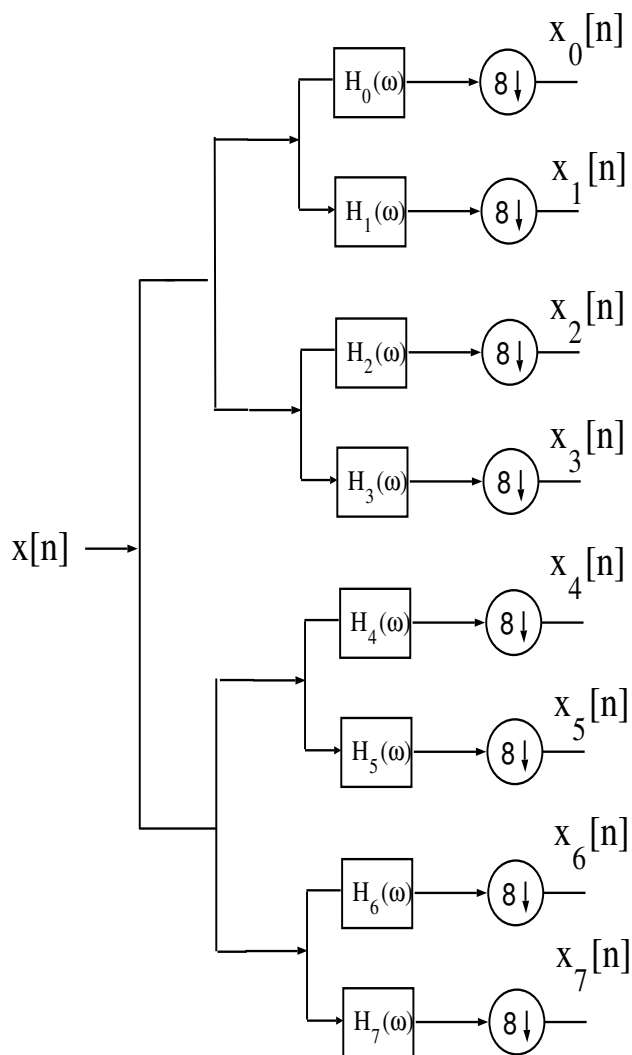


Figure 1(a). Analysis Filter Bank, $M = 8$.

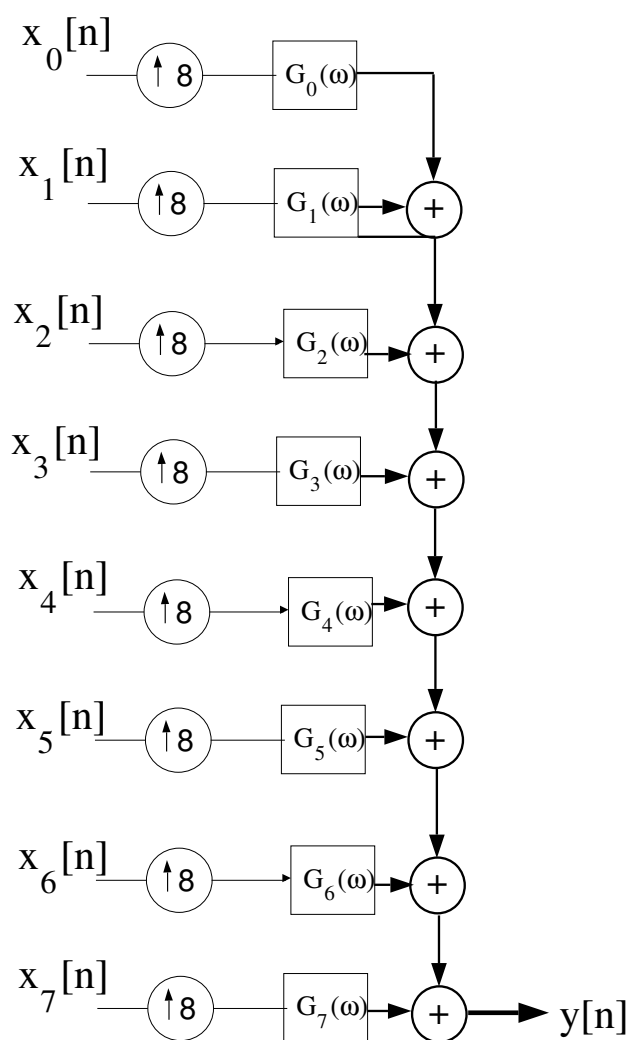


Figure 1(b). Synthesis Filter Bank, $M = 8$.

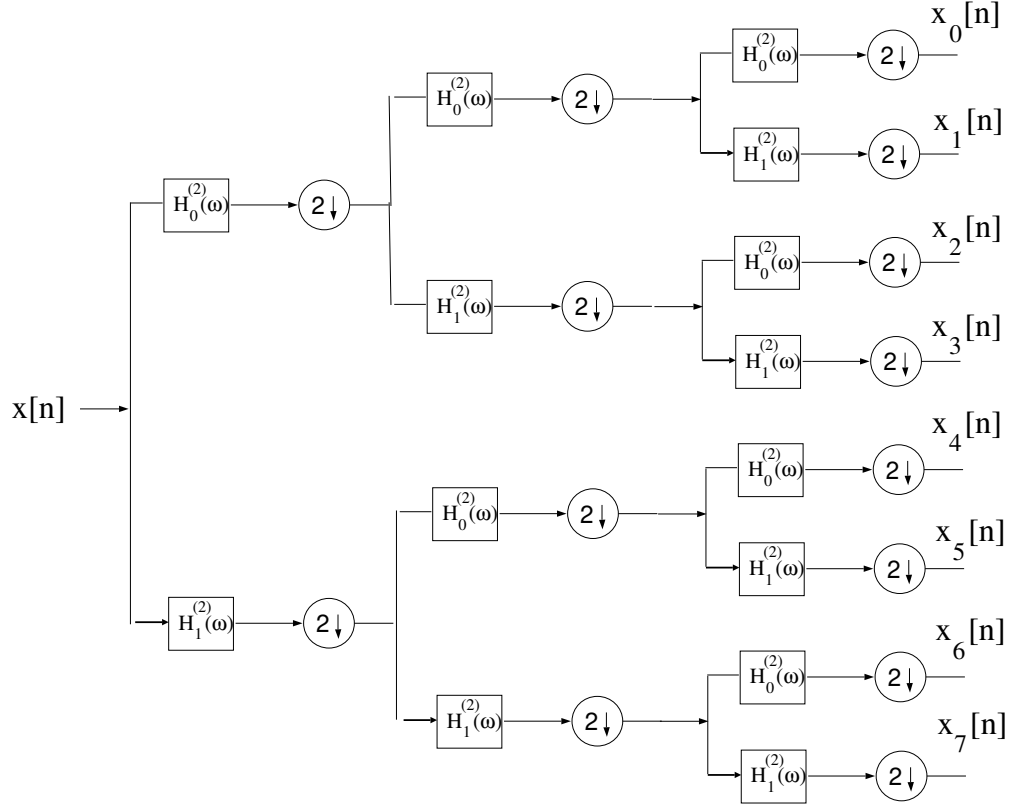


Figure 2(a). Analysis Section of Three-Stage Tree-Structured Filter Bank.

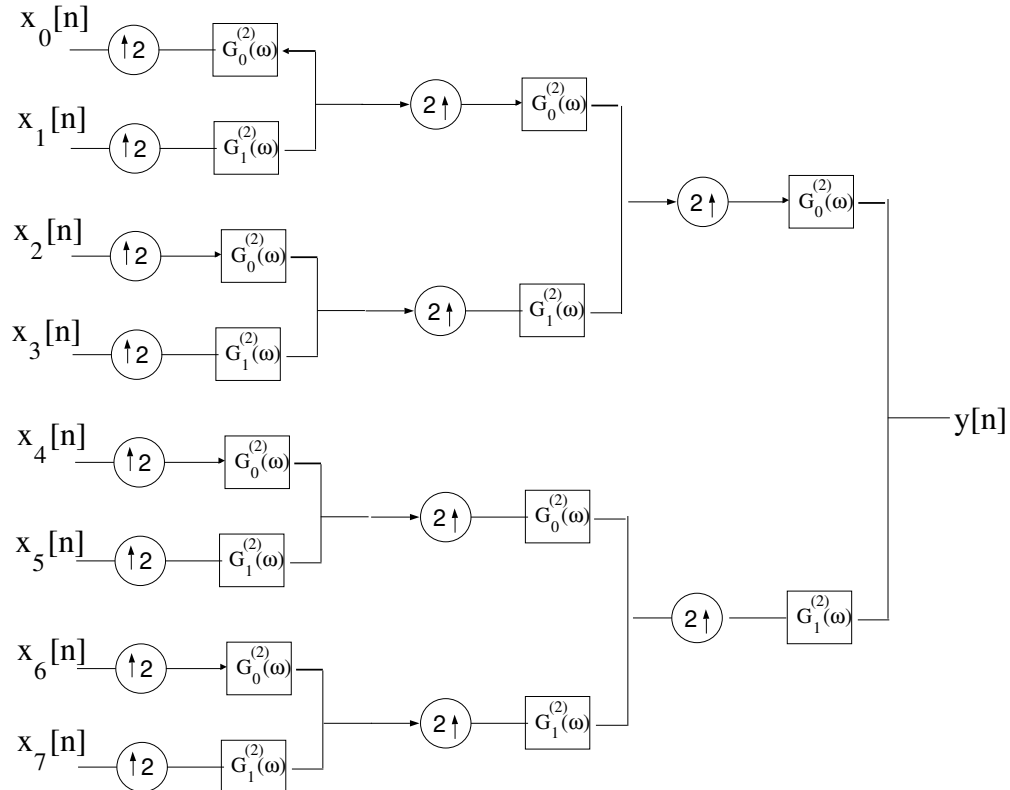


Figure 2(b). Synthesis Section of Three-Stage Tree-Structured Filter Bank.

Synthesizing M=8 Channel Perfect Reconstruction Filter Banks from Tree-Structured Filter Banks. This Matlab assignment is centered on synthesizing an M=8 channel uniform PR filter bank from a three stage tree-structured PR filter bank. As discussed in class, if the number of channels, M , is a power of two, an M-channel uniform PR filter bank may be synthesized via an equivalent (same I/O relationship) tree-structured PR filter bank having $\log_2(M)$ stages with each stage formed from a two-channel QMF filter bank as depicted in Figure 2, i.e., the combination of the analysis filter pair, $\{H_0^{(2)}(\omega), H_1^{(2)}(\omega)\}$, and synthesis filter pair $\{G_0^{(2)}(\omega), G_1^{(2)}(\omega)\}$, form a 2-channel PR filterbank.

Part I. Deriving the Uniform Filter Bank Equivalent to Tree-Structured Filter Bank. Using Noble's Decimation Identity to express each analysis filter, $H_m(\omega)$, $m = 0, 1, \dots, 7$, in terms of $H_0^{(2)}(\omega)$ and $H_1^{(2)}(\omega)$. In each case, express the corresponding impulse response $h_m[n]$, $m = 0, 1, \dots, 7$, in terms of $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$.

Next, use Noble's Upsampling Identity to express each synthesis filter, $G_m(\omega)$, $m = 0, 1, \dots, 7$, in terms of $G_0^{(2)}(\omega)$ and $G_1^{(2)}(\omega)$. In each case, express the corresponding impulse response $g_m[n]$, $m = 0, 1, \dots, 7$, in terms of $g_0^{(2)}[n]$ and $g_1^{(2)}[n]$. Note $g_0^{(2)}[n] = h_0^{(2)}[n]$ and $g_1^{(2)}[n] = -h_1^{(2)}[n]$; this is assumed throughout.

Part II. Matlab Calculations. For each pair of $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$ specified below, use Matlab and the results derived above to compute the numerical values of the analysis filters $h_m[n]$, $n = 0, 1, \dots, N-1$, for $m = 0, 1, \dots, 7$. Plot all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, \dots, 7$, superimposed on a single graph using a 4096 pt. FFT of each $h_m[n]$, $m = 0, 1, \dots, 7$. Next, compute the numerical values of the synthesis filters $g_m[n]$, $n = 0, 1, \dots, N-1$, for $m = 0, 1, \dots, 7$. Also, using Matlab, for each case below place each impulse response $h_m[n]$, $n = 0, 1, \dots, 7$, as the row of a matrix called **H** and compute **HH^H**. Note superscript H denotes Conjugate-Transpose or Hermitian-Transpose; in Matlab, it's tic-mark. The matrix **HH^H** contains the inner product between every pair of two filters; it will be a scalar multiple of the Identity Matrix if the filters are orthogonal. Put the elements of the resulting 8×8 matrix in a Table. Finally, generate a sample function of Gaussian random process with zero mean and a variance of 1, of length 128 as the input signal $x[n]$. Plot the magnitude of the DTFT of $x[n]$ using a 4096 pt. FFT; also plot the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank $y[n]$, using a 4096 pt. FFT.

(A) $h_0^{(2)}[n] = \frac{1}{\sqrt{2}}\{1, 1\}$ and $h_1^{(2)}[n] = \frac{1}{\sqrt{2}}\{1, -1\}$.

- (i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, \dots, 7$ *superimposed* as Figure 1(a).
- (ii) Label the table containing the values of the 8×8 matrix **HH^H** as Table 1.
- (i) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 1(b).
- (iv) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 1(c).

(B) $h_0^{(2)}[n] = h_{sr}[n-16]$, $n = 0, 1, \dots, 31$, $h_1^{(2)}[n] = (-1)^n h_0^{(2)}[n]$, and $\beta = 0.35$ where

$$h_{sr}[n] = \sqrt{2} \left\{ \frac{2\beta \cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]} \right\}, n = -16, \dots, 1, \dots, 15.$$

- (i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, \dots, 7$ *superimposed* as Figure 2(a).
 - (ii) Label the table containing the values of the 8×8 matrix $\mathbf{H}\mathbf{H}^H$ as Table 2.
 - (iii) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 2(b).
 - (iv) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 2(c).
- (C) $h_0^{(2)}[n] = h_{sr}[n - 24]$, $n = 0, 1, \dots, 47$, $h_1^{(2)}[n] = (-1)^n h_0^{(2)}[n]$, and $\beta = 0.1$ where
- $$h_{sr}[n] = \sqrt{2} \left\{ \frac{2\beta \cos[(1 + \beta)\pi(n + .5)/2]}{\pi[1 - 4\beta^2(n + .5)^2]} + \frac{\sin[(1 - \beta)\pi(n + .5)/2]}{\pi[(n + .5) - 4\beta^2(n + .5)^3]} \right\}, n = -24, \dots, 1, \dots, 23.$$
- (i) Label the plot of all of the corresponding DTFT's $H_m(\omega)$, $m = 0, 1, \dots, 7$ *superimposed* as Figure 3(a).
 - (iii) Label the table containing the values of the 8×8 matrix $\mathbf{H}\mathbf{H}^H$ as Table 3.
 - (iv) Label the plot of the magnitude of the DTFT of the Gaussian random process input signal as Figure 3(b).
 - (v) Label the plot of the magnitude of the DTFT of the corresponding output of the M=8 channel uniform PR filter bank as Figure 3(c).

General Information.

Deliverables for this project include:

- The Deliverables for Part I. You can show the work for just one of the 8 "chains" on both the analysis side and the synthesis side. On the analysis side, choose a chain that has at least one instance of $H_0^{(2)}(\omega)$ and one instance of $H_1^{(2)}(\omega)$. On the synthesis side, choose a chain that has at least one instance of $G_0^{(2)}(\omega)$ and one instance of $G_1^{(2)}(\omega)$.
- 9 plots and 3 tables
- a paragraph summarizing your observations and any conclusions you can draw from this set of computer experiments.
- your source code appended to the report

The collection of plots and accompanying explanation should be put together in a cohesive manner in the form of a brief report. You may use any Matlab command you like in solving these problems. Each student is expected to do his/her own work and each must turn in his/her own report. Again, your write-up for this homework should be in the form of a brief report. Handwriting is acceptable but please be sure it is legible.