7.29 *Frequency-domain sampling* The signal $x(n) = a^n$, $-1 < a < 1$ has a Fourier transform

$$X(\omega) = \frac{1 - a^2}{1 - 2a \cos \omega + a^2}$$

(a) Plot $X(\omega)$ for $0 \leq \omega \leq 2\pi$, $a = 0.8$. Reconstruct and plot $X(\omega)$ from its samples $X(2\pi k/N)$, $0 \leq k \leq N - 1$ for $N = 20$

(b) $N = 100$

(c) $N = 100$

(d) Compare the spectra obtained in parts (b) and (c) with the original spectrum $X(\omega)$ and explain the differences.

(e) Illustrate the time-domain aliasing when $N = 20$.

7.30 *Frequency analysis of amplitude-modulated discrete-time signal* The discrete-time signal

$$x(n) = \cos 2\pi f_1 n + \cos 2\pi f_2 n$$

where $f_1 = \frac{1}{18}$ and $f_2 = \frac{5}{128}$, modulates the amplitude of the carrier

$$x_c(n) = \cos 2\pi f_c n$$

where $f_c = \frac{30}{128}$. The resulting amplitude-modulated signal is

$$x_{am}(n) = x(n) \cos 2\pi f_c n$$

(a) Sketch the signals $x(n)$, $x_c(n)$, and $x_{am}(n)$, $0 \leq n \leq 255$.

(b) Compute and sketch the 128-point DFT of the signal $x_{am}(n)$, $0 \leq n \leq 127$.

(c) Compute and sketch the 128-point DFT of the signal $x_{am}(n)$, $0 \leq n \leq 99$.

(d) Compute and sketch the 256-point DFT of the signal $x_{am}(n)$, $0 \leq n \leq 179$.

(e) Explain the results obtained in parts (b) through (d), by deriving the spectrum of the amplitude-modulated signal and comparing it with the experimental results.

7.31 The sawtooth waveform in Fig. P7.31 can be expressed in the form of a Fourier series as

$$x(t) = \frac{2}{\pi} \left( \sin \pi t - \frac{1}{2} \sin 2\pi t + \frac{1}{3} \sin 3\pi t - \frac{1}{4} \sin 4\pi t \cdots \right)$$

(a) Determine the Fourier series coefficients $c_k$.

(b) Use an $N$-point subroutine to generate samples of this signal in the time domain using the first six terms of the expansion for $N = 64$ and $N = 128$. Plot the signal $x(t)$ and the samples generated, and comment on the results.