

• Example.

$$y[n] = \frac{13}{4} y[n-1] - \frac{3}{4} y[n-2] + x[n]$$

• Determine all possible impulse responses

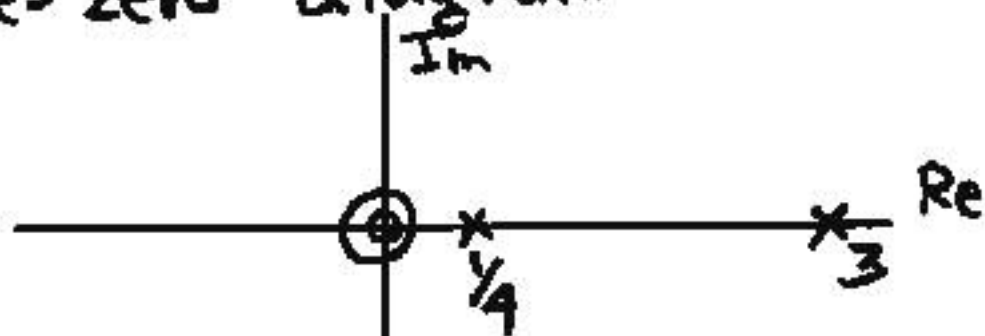
$$Y(z) = \frac{13}{4} z^{-1} Y(z) - \frac{3}{4} z^{-2} Y(z) + X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{13}{4} z^{-1} + \frac{3}{4} z^{-2}}$$

$$H(z) = \frac{z^2}{z^2 - \frac{13}{4} z + \frac{3}{4}} =$$

$$H(z) = \frac{z^2}{(z - \frac{1}{4})(z - 3)}$$

• pole-zero diagram:



Three possibilities for ROC

I. $|z| < \frac{1}{4}$

II. $\frac{1}{4} < |z| < 3$

III. $|z| > 3$

$$H(z) = A_1 \frac{z}{z - \frac{1}{4}} + A_2 \frac{z}{z - 3}$$

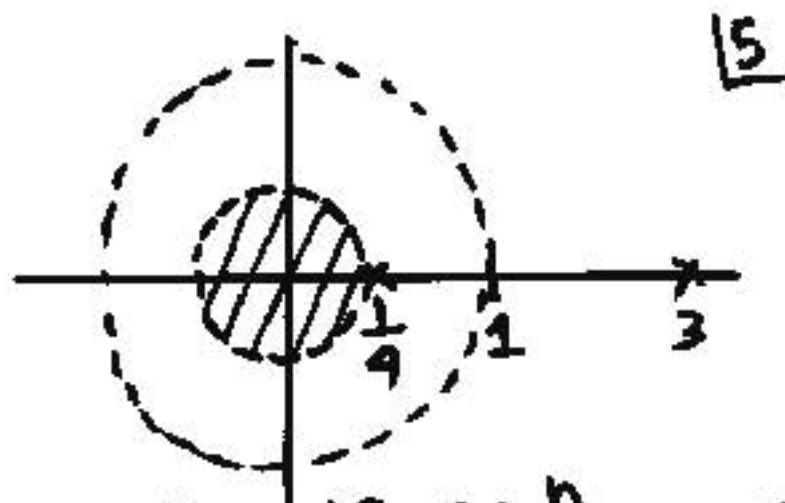
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$$A_1 = \left. \frac{z - \frac{1}{4}}{z} H(z) \right|_{z = \frac{1}{4}} =$$
$$= \left. \frac{z}{z - 3} \right|_{z = \frac{1}{4}} = -\frac{1}{11}$$

$$A_2 = \frac{12}{11}$$

$$H(z) = -\frac{1}{11} \frac{z}{z - \frac{1}{4}} + \frac{12}{11} \frac{z}{z - 3}$$

$$I. |z| < \frac{1}{4}$$



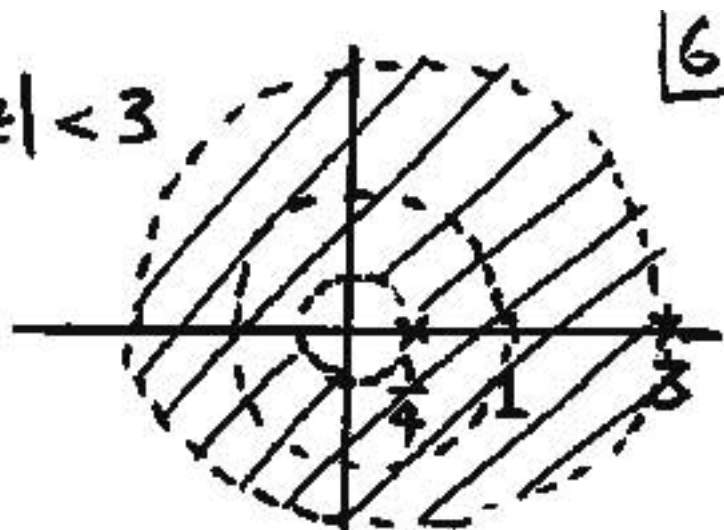
$$h[n] = \frac{1}{11} \left(\frac{1}{4}\right)^n u[-n-1] - \frac{12}{11} (3)^n u[-n-1]$$

• Causal? No! Left-sided sequence

• BIBO Stability? No!

$$\lim_{n \rightarrow -\infty} |h[n]| = +\infty$$

II. ROC: $\frac{1}{4} < |z| < 3$ 6

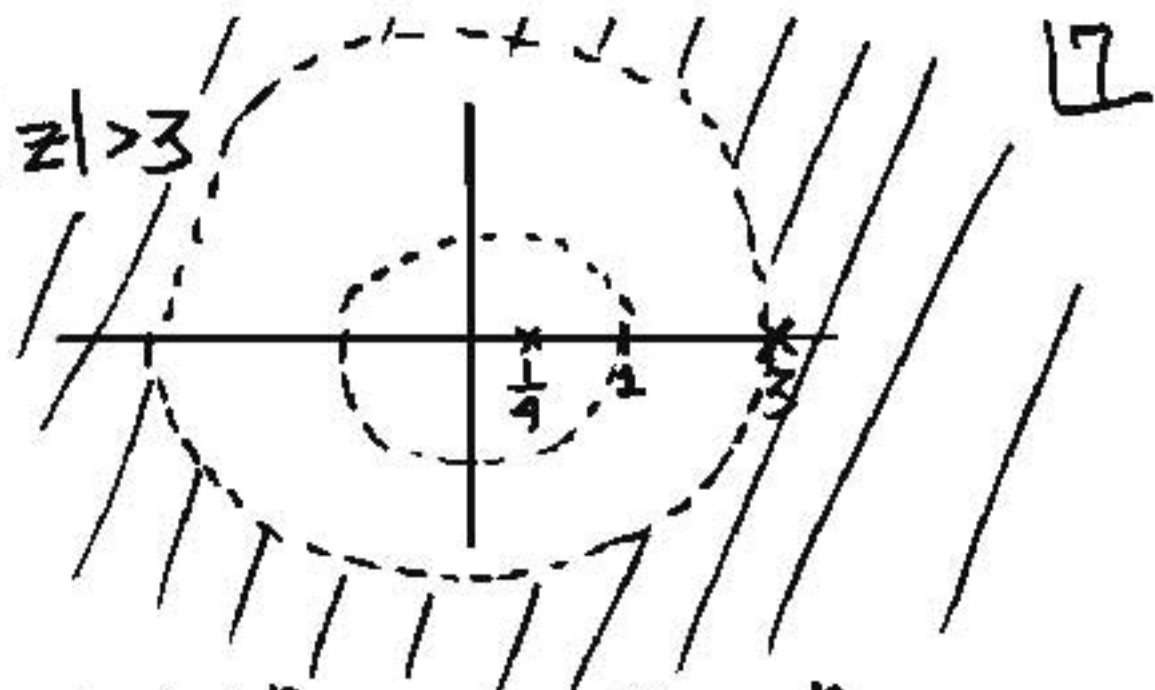


$$h[n] = \frac{-1}{11} \left(\frac{1}{4}\right)^n u[n] - \frac{12}{11} (3)^n u[-n-1]$$

• Causal? No! Two-Sided Sequence

• Stability? Yes! $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

III. $|z| > 3$



$$h[n] = \frac{1}{11} \left(\frac{1}{4}\right)^n u[n] + \frac{12}{11} (3)^n u[n]$$

- Causal? Yes!
- Stable? No!

- each of these three sol'n's. | 8
- for the impulse response satisfy

$$h[n] = \frac{13}{4}h[n-1] - \frac{3}{4}h[n-2] + \delta[n]$$

- only the causal sol'n. has practical value
- End of Example