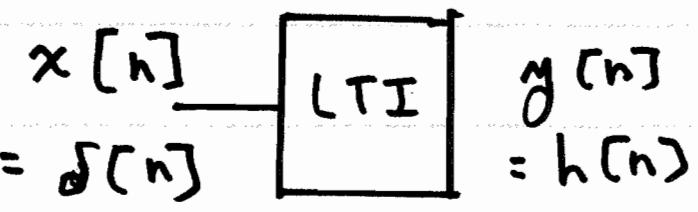
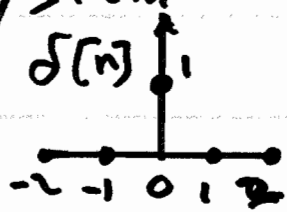


①
• Additional Features of Impulse Response

of LTI System



• BIBO Stability \Rightarrow necessary condition:

If: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$, then system is stable BIBO

Proof: $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

Bounded input: $|x[n]| < A_{\max, in} < \infty \quad \forall n$

Triangle Inequality: $|a + b| \leq |a| + |b|$

Thus: $|y[n]| < \sum_k |h[k] x[n-k]| = \sum_k |h[k]| |x[n-k]|$

• Since $|x[n-k]| < A_{\max, in} \quad \forall n, k$, it follows (2)

$$|y[n]| < \left\{ \sum_{k=-\infty}^{\infty} |h[k]| \right\} A_{\max, in}$$

Thus, if $B = \sum_{k=-\infty}^{\infty} |h[k]| < \infty$, then $|y[n]| < \infty$

Note: IF $\sum_{k=-\infty}^{\infty} |h[k]| = \infty$ and you select input

as $x[n] = \frac{h^*[-n]}{|h[-n]|}$: $y[n] = \sum_{k=-\infty}^{\infty} h[k] \frac{h^*[-(n-k)]}{|h[n-k]|}$

$|x[n]| = 1 \quad \forall n$ \nearrow

Consider $y[0]$:

$$y[0] = \sum_{k=-\infty}^{\infty} \frac{h[k] h^*[k]}{|h[k]|} = \sum_{k=-\infty}^{\infty} \frac{|h[k]|^2}{|h[k]|} = \sum_{k=-\infty}^{\infty} |h[k]| = \infty$$

So, it is a necessary condition for stability

• Causality: $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] =$ (3)

... $+ h[-2] x[n+2] + h[-1] x[n+1] + h[0] x[n] +$
 $+ h[1] x[n-1] + h[2] x[n-2] + \dots$

• Thus, if $h[n] = 0$ for $n < 0$, then system is causal \Rightarrow does not depend on future inputs

• For CT LTI Systems:

• BIBO Stability: $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

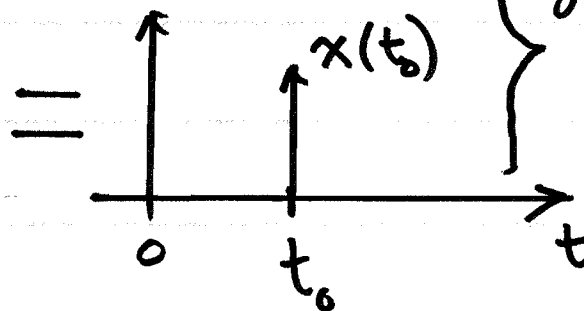
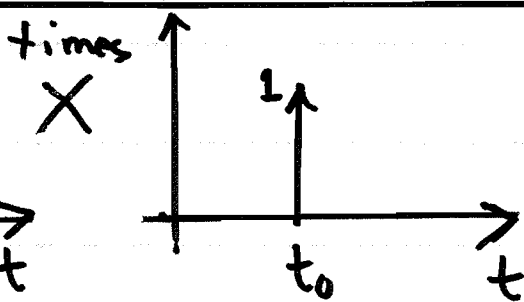
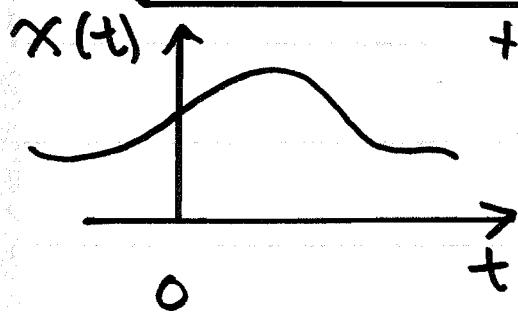
• Causality: $h(t) = 0$ for $t < 0$

• Recall properties of (Dirac) Delta Function: (4)

Area: $\int_{-\infty}^{\infty} \delta(t) dt = 1$

$$\int_{-\infty}^{\infty} \delta(t-t_0) dt = \int_{t_0^-}^{t_0^+} \delta(t-t_0) dt = 1$$

$$\boxed{x(t) \delta(t-t_0) = x(t_0) \delta(t-t_0)}$$



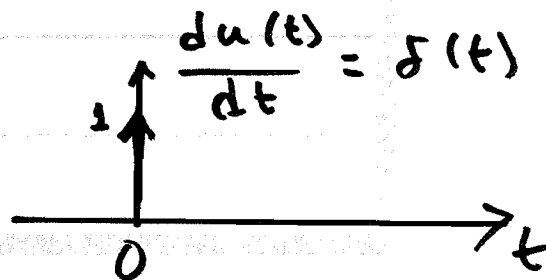
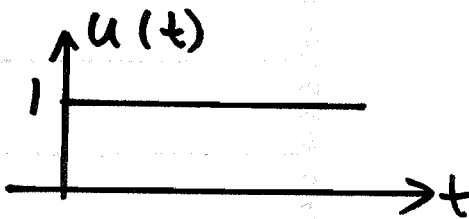
Multiplication
by a Delta
Function

Symmetry:

$$\boxed{\delta(-t) = \delta(t)}$$

Derivative at
discontinuity:

$$\boxed{\delta(t) = \frac{du(t)}{dt}}$$



• Additional Properties of Delta Function:

5

$$x(t) * \delta(t) = x(t)$$

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

Proof: $x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau-t_0) x(t-\tau) d\tau$

• multiplication
Property dictates

$$= \int_{-\infty}^{\infty} \delta(\tau-t_0) x(t-t_0) d\tau$$

• $x(t-t_0)$ does not
depend on integration
variable τ

$$= x(t-t_0) \int_{-\infty}^{\infty} \delta(\tau-t_0) d\tau$$

• area under Delta
Function is unity

$$= x(t-t_0)$$

Can easily
show:

$$\delta(t-t_1) * \delta(t-t_2) = \delta(t-(t_1+t_2))$$