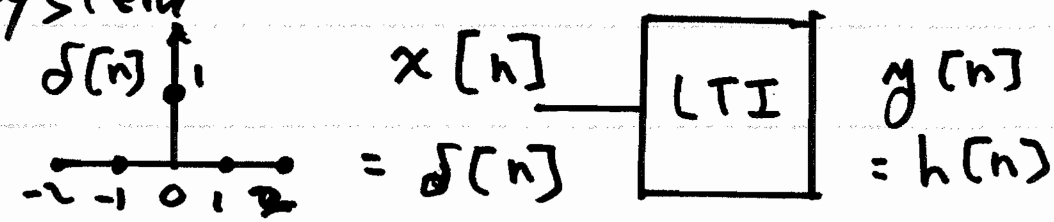
- Additional Features of Impulse Response of LTI System



- BIBO Stability ⇒ necessary condition:

If: $$\sum_{n=-\infty}^{\infty} |h[n]| < \infty,$$ then system is BIBO stable

<u>Proof:</u> $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Bounded input: $|x[n]| < A_{max_{in}} < \infty \quad \forall n$

Triangle Inequality: $|a + b| \leq |a| + |b|$

<u>Thus:</u> $$|y[n]| < \sum_{k} |h[k]\, x[n-k]| = \sum_{k} |h[k]|\, |x[n-k]|$$

- Since $|x[n-k]| < A_{max_{in}}$ $\forall n$ $\forall k$, it follows

$$|y[n]| < \left\{ \sum_{k=-\infty}^{\infty} |h[k]| \right\} A_{max_{in}}$$

Thus, if $B = \sum_{k=-\infty}^{\infty} |h[k]| < \infty$, then $|y[n]| < \infty$

Note: If $\sum_{k=-\infty}^{\infty} |h[k]| = \infty$ and you select input

as $x[n] = \dfrac{h^*[-n]}{|h[-n]|}$; $y[n] = \sum_{k=-\infty}^{\infty} h[k] \dfrac{h^*[-(n-k)]}{|h[-(n-k)]|}$

$\boxed{|x[n]| = 1 \ \forall n}$

Consider $y[0]$:

$$y[0] = \sum_{k=-\infty}^{\infty} \frac{h[k]\, h^*[k]}{|h[k]|} = \sum_{k=-\infty}^{\infty} \frac{|h[k]|^2}{|h[k]|} = \sum_{k=-\infty}^{\infty} |h[k]| = \infty$$

So, it is a <u>necessary</u> condition for stability

- Causality: $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] =$

$$\ldots + h[-2]\,x[n+2] + h[-1]\,x[n+1] + h[0]\,x[n] +$$

$$+ h[1]\,x[n-1] + h[2]\,x[n-2] + \ldots$$

- Thus, if $h[n] = 0$ for $n < 0$, then system is causal $\Rightarrow$ does <u>not</u> depend on future inputs

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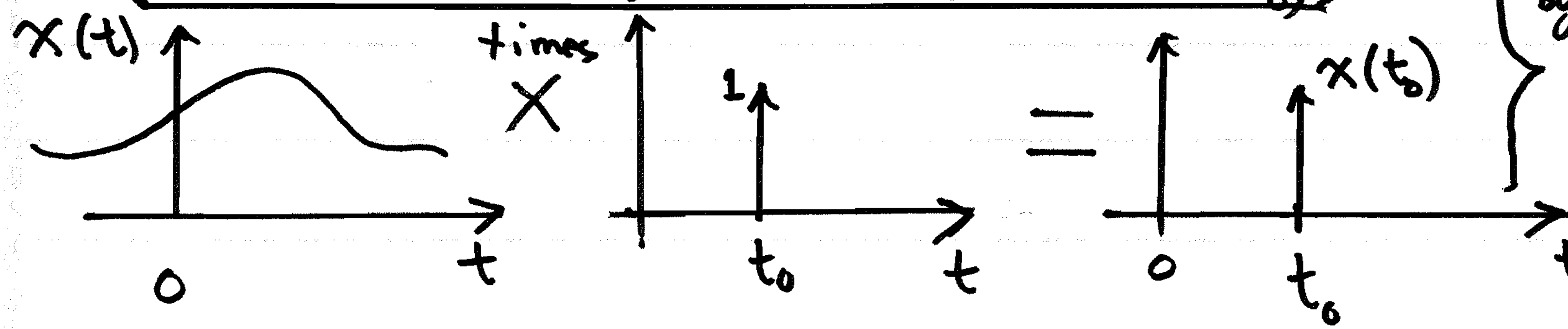
- For CT LTI Systems:
- <u>BIBO Stability</u>: $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$
- <u>Causality</u>: $h(t) = 0$ for $t < 0$

- Recall properties of (Dirac) Delta Function:

<u>Area:</u> $$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad \int_{-\infty}^{\infty} \delta(t-t_0) = \int_{t_0^-}^{t_0^+} \delta(t-t_0)\,dt = 1$$

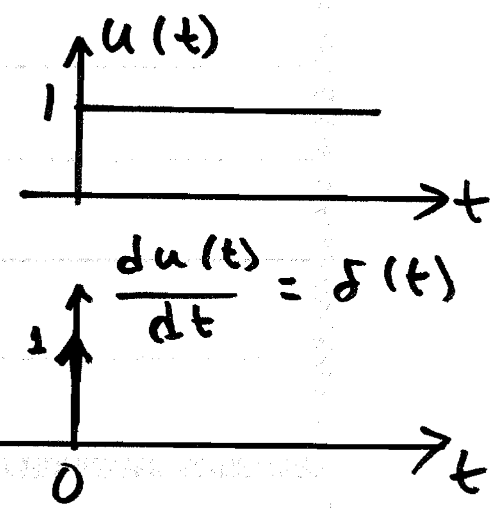
$$\boxed{x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)}$$



Multiplication by a Delta Function

<u>Symmetry:</u> $$\boxed{\delta(-t) = \delta(t)}$$

<u>Derivative at discontinuity</u>: $$\boxed{\delta(t) = \frac{du(t)}{dt}}$$



- Additional Properties of Delta Function:

$$\boxed{x(t) * \delta(t) = x(t)} \qquad \boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

Proof: $x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-\tau)\, d\tau$

- multiplication property dictates

$$= \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-t_0)\, d\tau$$

- $x(t-t_0)$ does not depend on integration variable τ

$$= x(t-t_0) \int_{-\infty}^{\infty} \delta(\tau - t_0)\, d\tau$$

- area under Delta Function is unity

$$= x(t-t_0)$$

Can easily show:

$$\delta(t-t_1) * \delta(t-t_2) = \delta\big(t-(t_1+t_2)\big)$$

Convolution with a DT Kronecker Delta

$$y[n] = x[n] * \delta[n-n_0] = x[n-n_0]$$

<u>Proof</u>: $$y[n] = \sum_{k=-\infty}^{\infty} \underbrace{\delta[k-n_0]}_{\substack{=1 \quad k=n_0 \\ 0 \quad \text{otherwise}}} x[n-k]$$

$\underbrace{\text{for all other values of } k}_{\text{so only 1 nonzero term in sum}}$

$$= x[n-n_0]$$