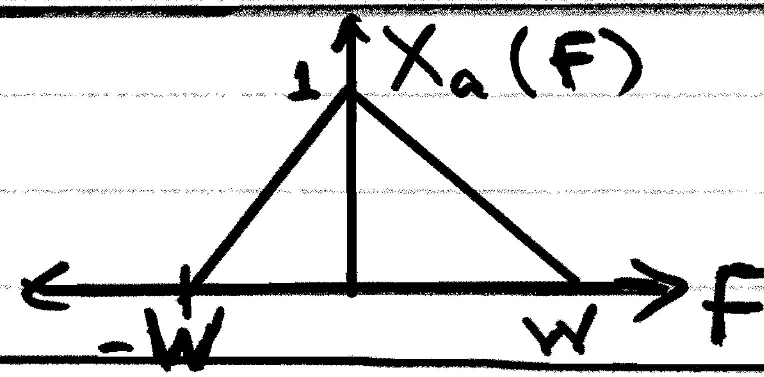
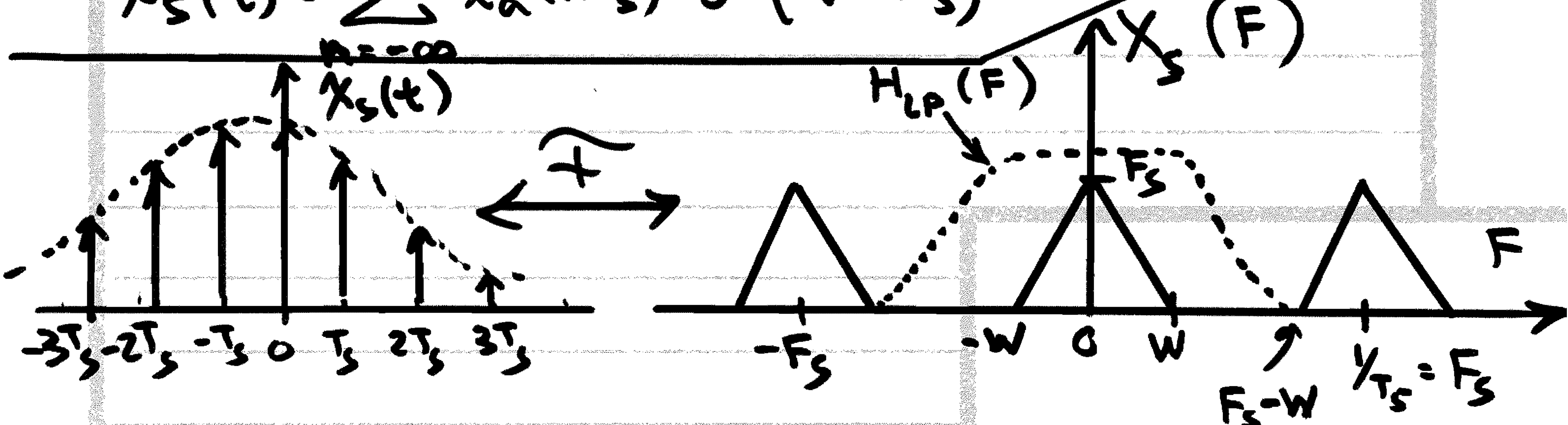
Quick Overview of Ideal D/A Conversion

$x_a(t) \xleftrightarrow{\mathcal{F}}$



$$x_s(t) = \sum_{n=-\infty}^{\infty} x_a(nT_s)\,\delta(t-nT_s) = x_a(t)\sum_{n}\delta(t-nT_s)$$



To avoid aliasing $F_s - W > W$

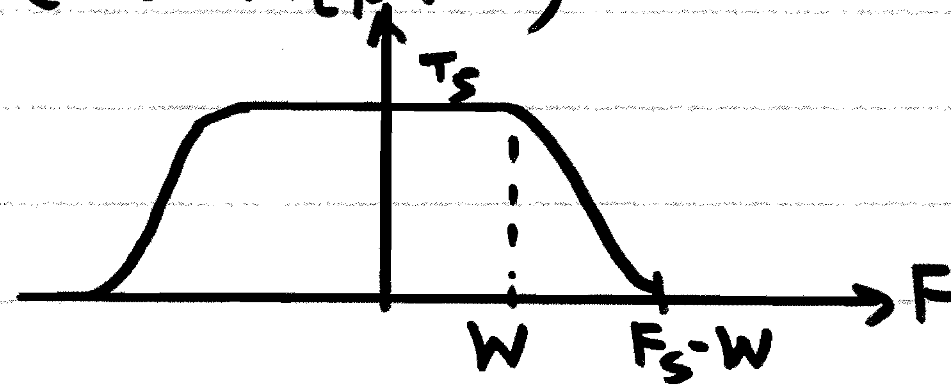
$\Rightarrow$ $F_s > 2W$ $\Rightarrow$ If $F_s \gg 2W$ (oversampling)

there is a don't care region between W and $F_s - W$

$\Rightarrow$ don't need Ideal LPF

THUS: $x_a(t) = x_s(t) * h_{LP}(t)$

where: $h_{LP}(t) \xleftrightarrow{\mathcal{F}} H_{LP}(F)$



$$x_s(t) = \sum_{n=-\infty}^{\infty} \underbrace{x_a(nT_s)}_{x[n]} \delta(t - nT_s) \longrightarrow \boxed{h_{LP}(t)} \longrightarrow \overset{\text{reconstructed}}{x_r(t)} = x_a(t)$$

Distributive Property of Convolution:

$$x_s(t) * h_{LP}(t) = \sum_{n=-\infty}^{\infty} x_a(nT_s) \left\{ \delta(t - nT_s) * h_{LP}(t) \right\}$$

$$= \sum_{n=-\infty}^{\infty} x_a(nT_s)\, h_{LP}(t - nT_s)$$

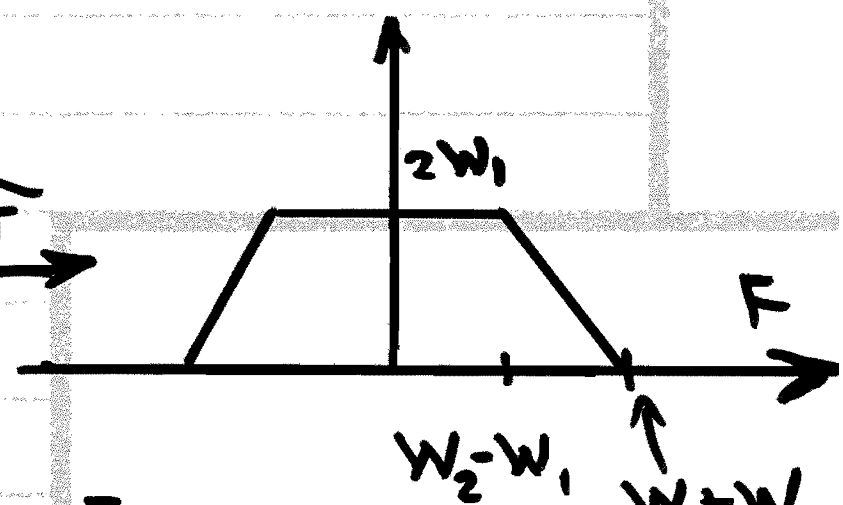
Possibilities for $h_{LP}(t)$:

$$h_{LP}(t) = T_s \frac{\sin(2\pi F_c t)}{\pi t} \quad \text{where: } W < F_c < F_s - W$$

$$h_{LP}(t) \propto \frac{\sin\left[\left(\frac{F_s}{2} - W\right) 2\pi t\right]}{\pi t} \frac{\sin\left[2\pi \frac{F_s}{2} t\right]}{\pi t}$$

Note:

$$\frac{\sin(2\pi W_1 t)}{\pi t} \frac{\sin(2\pi W_2 t)}{\pi t} \xleftrightarrow{\mathcal{F}}$$



$W_2 > W_1$

THUS: with $W_1 = \frac{F_s}{2} - W$ and $W_2 = \frac{F_s}{2}$

$W_2 - W_1 = W$ and $W_1 + W_2 = F_s - W$

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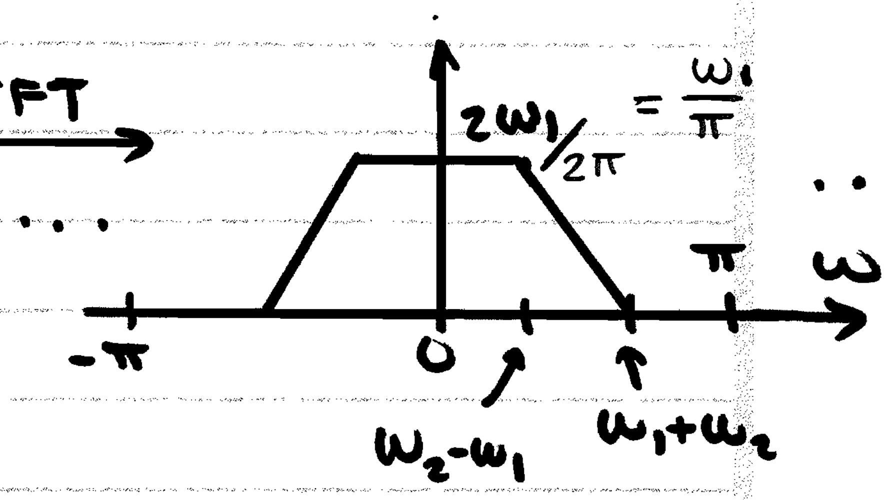
Note: for future exams:

in Discrete-Time:

$$\frac{\sin(\omega_1 n)}{\pi n} \, \frac{\sin(\omega_2 n)}{\pi n} \overset{DTFT}{\longleftrightarrow}$$



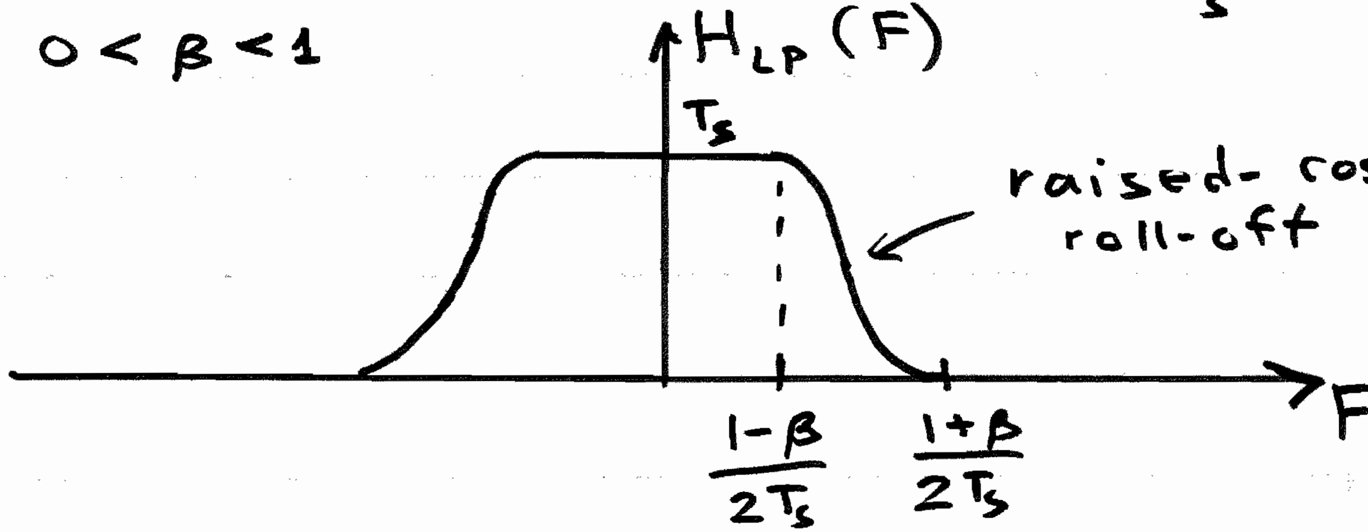
$\omega_1 < \omega_2$

$\omega_1 + \omega_2 < \pi$

• Another possibility for $h_{LP}(t)$:

$$h_{LP}(t) = T_s \frac{\sin\left(\pi \frac{t}{T_s}\right)}{\pi \frac{t}{T_s}} \cdot \frac{\cos\left(\pi \beta \frac{t}{T_s}\right)}{1 - 4\beta^2 \frac{t^2}{T_s^2}}$$

• for $0 < \beta < 1$



$$\left.\begin{aligned} \frac{1+\beta}{2T_s} &= F_s - W = \frac{1}{T_s} - W \\ \frac{1-\beta}{2T_s} &= W \end{aligned}\right\} \text{Take difference} \Rightarrow \frac{\beta}{T_s} = \frac{1}{T_s} - 2W$$

$$\Rightarrow \beta = 1 - 2WT_s \Rightarrow \beta = 1 - \frac{2W}{F_s}$$