

EE 538 DSP I Exam 1 Sol'n. F'99
Sol'n. to Prob. 1

Sol'n. : cross-correlate with a user's code.
Since code length is 2, take every other value and divide by 2.

$$r_{xc_1}[l] = \{0, -2, -4, -2, 2, 2, 0, 2\}$$

$$b_1[n] = \frac{1}{2} r_{xc_1}[2n+1]$$

$$= \frac{1}{2} \{-2, -2, 2, 2\} =$$

$$= \{-1, -1, 1, 1\} \leftarrow \text{ANS.}$$

$$r_{xc_2}[l] = \{0, 2, 0, -2, -2, 2, 0, -2, 2\}$$

$$b_2[n] = \frac{1}{2} r_{xc_2}[2n+1]$$

$$= \frac{1}{2} \{2, -2, 2, -2\}$$

$$= \{1, -1, 1, -1\} \leftarrow \text{ANS.}$$

(A) $y[n] = r \sin \theta y_1[n-1] + r \cos \theta y[n-1]$

(B) $y_1[n] = x[n] - r \sin \theta y[n-1] + r \cos \theta y_1[n-1]$

• ZT of B:

$$Y_1(z) \{1 - r \cos \theta z^{-1}\} = X(z) - r \sin \theta z^{-1} Y(z)$$

(B) $Y_1(z) = \left\{ \frac{1}{1 - r \cos \theta z^{-1}} X(z) - \frac{r \sin \theta z^{-1}}{1 - r \cos \theta z^{-1}} Y(z) \right\}$

• ZT of A:

(A) $Y(z) = r \sin \theta z^{-1} Y_1(z) + r \cos \theta z^{-1} Y(z)$

$$= \frac{r \sin \theta z^{-1}}{1 - r \cos \theta z^{-1}} X(z) - \frac{r^2 \sin^2 \theta z^{-2}}{1 - r \cos \theta z^{-1}} Y(z)$$

$$+ r \cos \theta z^{-1} Y(z)$$

$$Y(z) \left\{ \frac{1 - r \cos \theta z^{-1} + r^2 \sin^2 \theta z^{-2} + r \cos^2 \theta z^{-2}}{1 - r \cos \theta z^{-1}} \right\}$$

$$= \frac{r \sin \theta z^{-1}}{1 - r \cos \theta z^{-1}} X(z)$$

EE538 DSP I Exam 1 Sol'n, F'99
Sol'n. to Prob. 2 (cont.)

(3)

Ans to (a) $H(z) = \frac{Y(z)}{X(z)} = \frac{r \sin \theta z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$

$\Rightarrow \frac{r \sin \theta z}{z^2 - 2r \cos \theta z + r^2}$

$H(z) = \frac{r \sin \theta z}{(z - r e^{j\theta})(z - r e^{-j\theta})}$

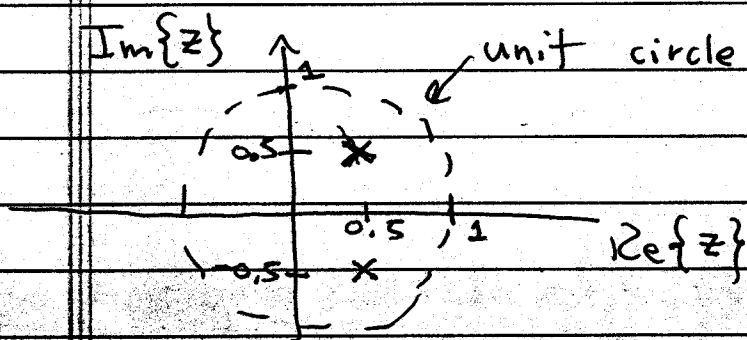
(b) poles are $r e^{j\theta}$ and $r e^{-j\theta}$

BIBO stability requires $r < 1$ ($|r| < 1$)

(c) when $r = \frac{1}{\sqrt{2}}$ and $\theta = 45^\circ$, poles are

$P_1 = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}} = \frac{1}{2} + \frac{j}{2}$

$P_2 = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} = \frac{1}{2} - \frac{j}{2}$



ROC: $|z| > \frac{1}{\sqrt{2}}$
 since causal

Sol'n. to Prob. (2) (cont.)

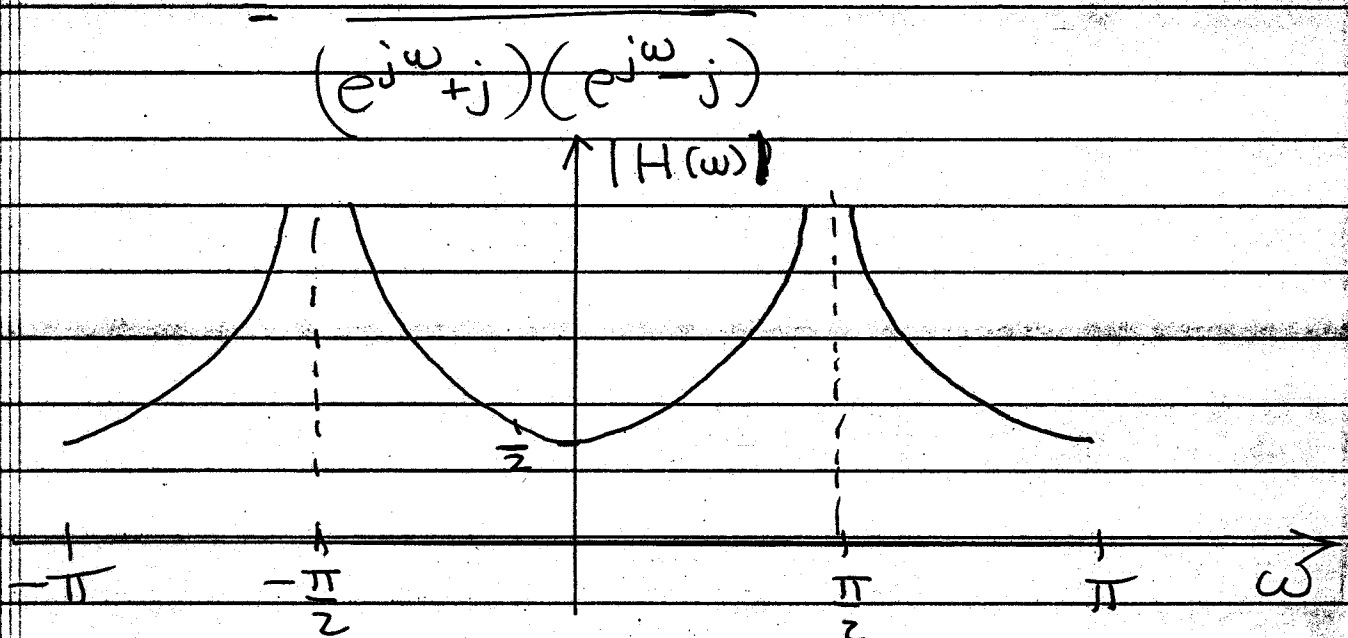
(4)

(d) with $r=1$ and $\theta=90^\circ$, the poles are

$$p_1 = e^{j\pi/2} = j \quad \text{and} \quad p_2 = e^{-j\pi/2} = -j$$

$$H(\omega) = H(z) \Big|_{z=e^{j\omega}} = \frac{z}{(z-j)(z+j)} = \frac{e^{j\omega}}{(e^{j\omega}-j)(e^{j\omega}+j)}$$

(i)



$$(ii) \quad X[n] = e^{j\frac{\pi}{2}n} \quad \text{or} \quad X[n] = \cos\left(\frac{\pi}{2}n\right)$$

$$\text{Since} \quad \lim_{\omega \rightarrow \frac{\pi}{2}} |H(\omega)| = \infty$$

$$y(t) = x(t) * g(t)$$

$$= x(t) * \{ \delta(t) - \delta(t-\tau) \}$$

$$= x(t) - x(t-\tau)$$

$$= \sum_{k=-\infty}^{\infty} b[k] \{ p(t-kT_0) - p(t-kT_0-\tau) \}$$

$$y[n] = \sum_{k=-\infty}^{\infty} b[k] \{ p(nT_0 - kT_0) - p(nT_0 - kT_0 - \tau) \}$$

for
 $\tau = T_0$

$$= \sum_{k=-\infty}^{\infty} b[k] \{ p[n-k] - p[n-k-1] \}$$

$$= \sum_{k=-\infty}^{\infty} b[k] h[n-k]$$

-where: $h[n] = p[n] - p[n-1]$

$$p[n] = p(nT_0) = \tilde{p}[2n] = 4\delta[n]$$

Answer to (a) $\left\{ \text{Thus: } h[n] = 4\delta[n] - 4\delta[n-1] \right\}$

Sol'n. to Prob. 3 (cont.)

(5)

for $\tau = \frac{T_0}{2}$:

$$y[n] = \sum_{k=-\infty}^{\infty} b[k] \left\{ p(nT_0 - kT_0) - p(nT_0 - kT_0 - \frac{T_0}{2}) \right\}$$

$$= \sum_{k=-\infty}^{\infty} b[k] \left\{ p(2(n-k)\frac{T_0}{2}) - p(2(n-k)-1)\frac{T_0}{2} \right\}$$

$$= \sum_{k=-\infty}^{\infty} b[k] h[n-k]$$

where: $h[n] = \tilde{p}[2n] - \tilde{p}[2n-1]$

$$= 4\delta[n] - \{1, -2, -2, 1\}$$

↑
n=0

$$= \{-1, 6, 2, -1\}$$

↑
n=0