My New Version of Problem. 2.65. pp. 144-145 of the Proakis and Manolakis Textbook. This is the model to simulate for all parts:

$$y[n] = x[n - 20] + a_2 x[n - D_2] + v[n], \quad n = 0, 1, ..., 199.$$  

where for every value of $n$, $v[n]$ is a zero-mean, independent, Gaussian random variable with a standard deviation of 1, for all parts.

For each of 3 different sequences,

(a) $x[n] = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1\} \ (M = 13)$

(b) $x[n] = \{-1, 1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1\} \ (M = 15)$

(c) $x[n]$ of length $M = 127$ generated according to shift-register defined in Prob. 2.65.

simulate 3 different values of the parameter pair $\{a_2, D_2\}$,

(1) $a_2 = 1, \quad D_2 = 22$

(2) $a_2 = 1, \quad D_2 = 21$

(3) $a_2 = -1, \quad D_2 = 21$

and do the following 3 plots.

(i) Plot the values of $x[n]$, for $n = 0, 1, ..., M - 1$, where $M$ is either 13, 15, or 127.

(ii) Plot the values of $y[n]$, for $n = 0, 1, ..., 199$.

(iii) Plot the cross-correlation $r_{y|x}(\ell)$, for $n = 0, 1, ..., 59$.

Put 3 plots per page so that there is a total of 9 pages of plots. Label each page with the values of $M$, $a_2$, and $D_2$. You can do either stem plots or line plots.

Page 1: $a_2 = 1, \quad D_2 = 22, \quad M = 13$: do plots (i), (ii), and (iii)

Page 2: $a_2 = 1, \quad D_2 = 21, \quad M = 13$: do plots (i), (ii), and (iii)

Page 3: $a_2 = -1, \quad D_2 = 21, \quad M = 13$: do plots (i), (ii), and (iii)

Page 4: $a_2 = 1, \quad D_2 = 22, \quad M = 15$: do plots (i), (ii), and (iii)

Page 5: $a_2 = 1, \quad D_2 = 21, \quad M = 15$: do plots (i), (ii), and (iii)

Page 6: $a_2 = -1, \quad D_2 = 21, \quad M = 15$: do plots (i), (ii), and (iii)

Page 7: $a_2 = 1, \quad D_2 = 22, \quad M = 127$: do plots (i), (ii), and (iii)

Page 8: $a_2 = 1, \quad D_2 = 21, \quad M = 127$: do plots (i), (ii), and (iii)

Page 9: $a_2 = -1, \quad D_2 = 21, \quad M = 127$: do plots (i), (ii), and (iii)
Note 1: This homework is worth $15/3 = 5$ points of your final grade.

Note 2: The goal of this Matlab homework is to exercise you on the practical applications of discrete-time cross-correlation. An additional goal is to get you started on using Matlab.

**General Information.**

Deliverables for this project include 27 plots. Each plot should be clearly labeled, and should be accompanied by a brief explanation. The collection of plots and accompanying explanations should be put together in a cohesive manner in the form of a very brief report. Don’t go overboard – this is simply a homework, **not** a project. Append source code to the report.

You may use any Matlab command you like in solving these problems. Each student is expected to do his/her own work and each must turn in his/her own report. Again, your write-up for this homework should be in the form of a very brief report. Handwriting is acceptable but please be sure it is legible. Your report should include:

- Answers to all questions posed including mathematical development where necessary.
- The 27 plots and observations/explanations
2.62 Determine the autocorrelation sequences of the following signals.

(a) \( x(n) = [1, 2, 1, 1] \)

(b) \( y(n) = [1, 1, 2, 1] \)

What is your conclusion?

2.63 What is the normalized autocorrelation sequence of the signal \( x(n) \) given by

\[
x(n) = \begin{cases} 
1, & -N \leq n \leq N \\
0, & \text{otherwise}
\end{cases}
\]

2.64 An audio signal \( s(t) \) generated by a loudspeaker is reflected at two different walls with reflection coefficients \( r_1 \) and \( r_2 \). The signal \( x(t) \) recorded by a microphone close to the loudspeaker, after sampling, is

\[
x(n) = s(n) + r_1 s(n - k_1) + r_2 s(n - k_2)
\]

where \( k_1 \) and \( k_2 \) are the delays of the two echoes.

(a) Determine the autocorrelation \( r_{xx}(l) \) of the signal \( x(n) \).

(b) Can we obtain \( r_1, r_2, k_1, \) and \( k_2 \) by observing \( r_{xx}(l) \)?

(c) What happens if \( r_2 = 0 \)?

2.65 Time-delay estimation in radar

Let \( x_a(t) \) be the transmitted signal and \( y_a(t) \) be the received signal in a radar system, where

\[
y_a(t) = a x_a(t - t_d) + v_a(t)
\]

and \( v_a(t) \) is additive random noise. The signals \( x_a(t) \) and \( y_a(t) \) are sampled in the receiver, according to the sampling theorem, and are processed digitally to determine the time delay and hence the distance of the object. The resulting discrete-time signals are

\[
x(n) = x_a(nT) \\
y(n) = y_a(nT) = a x_a(nT - DT) + v_a(nT)
\]

\[
\equiv a x(n - D) + v(n)
\]

Figure P2.65

Linear feedback shift register.
(a) Explain how we can measure the delay $D$ by computing the crosscorrelation $r_{xy}(l)$.

(b) Let $x(n)$ be the 13-point Barker sequence

$$x(n) = \{+1, +1, +1, +1, +1, -1, -1, +1, +1, -1, -1, +1, +1\}$$

and $v(n)$ be a Gaussian random sequence with zero mean and variance $\sigma^2 = 0.01$. Write a program that generates the sequence $y(n)$, $0 \leq n \leq 199$ for $a = 0.9$ and $D = 20$. Plot the signals $x(n)$, $y(n)$, $0 \leq n \leq 199$.

(c) Compute and plot the crosscorrelation $r_{xy}(l)$, $0 \leq l \leq 59$. Use the plot to estimate the value of the delay $D$.

(d) Repeat parts (b) and (c) for $\sigma^2 = 0.1$ and $\sigma^2 = 1$.

(e) Repeat parts (b) and (c) for the signal sequence

$$x(n) = \{-1, -1, +1, +1, +1, +1, -1, -1, +1, +1, -1, -1\}$$

which is obtained from the four-stage feedback shift register shown in Fig. P2.65. Note that $x(n)$ is just one period of the periodic sequence obtained from the feedback shift register.

(f) Repeat parts (b) and (c) for a sequence of period $N = 2^7 - 1$, which is obtained from a seven-stage feedback shift register. Table 2.2 gives the stages connected to the modulo-2 adder for (maximal-length) shift-register sequences of length $N = 2^m - 1$.

**TABLE 2.2** Shift-Register Connections for Generating Maximal-Length Sequences

<table>
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<tr>
<th>$m$</th>
<th>Stages Connected to Modulo-2 Adder</th>
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</tr>
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