ECE 53800 Digital Signal Processing I **Homework 1**

Due Date for On-Campus Students: Turn in printout on Friday, 4 October 2019

Due Date for Off-Campus Students: Due by e-mail on Oct. 4.

My New Version of Problem. *2.65.* pp. 144-145 of the Proakis and Manolakis Textbook. This is the model to simulate for all parts:

$$y[n] = x[n-20] + a_2 x[n-D_2] + v[n], \quad n = 0, 1, ..., 199.$$

where for every value of n, $v[n]$ is a zero-mean, independent, Gaussian random variable with a standard deviation of 1, for all parts.
For each of 3 different sequences,

(a) $x[n] = \{1, 1, 1, 1, 1, -1 - 1, 1, 1, -1, 1, -1, 1\}$ $(M = 13)$

(b) $x[n] = \{-1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, 1, -1, -1, 1\}$ $(M = 15)$

(c) $x[n]$ of length $M = 127$ generated according to shift-register defined in Prob. 2.65.

simulate 3 different values of the parameter pair $\{a_2, D_2\}$,

(1) $a_2 = 1$, $D_2 = 22$

(2) $a_2 = 1$, $D_2 = 21$

(3) $a_2 = -1$, $D_2 = 21$

and do the following 3 plots.

(i) Plot the values of $x[n]$, for $n = 0, 1, ..., M-1$, where M is either 13, 15, or 127.

(ii) Plot the values of $y[n]$, for $n = 0, 1, ..., 199$.

(iii) Plot the cross-correlation $r_{yx}(\ell)$, for $n = 0, 1, ..., 59$.

Put 3 plots per page so that there is a total of 9 pages of plots. Label each page with the values of M, a_2, and D_2. You can do either stem plots or line plots.

Page 1: $a_2 = 1$, $D_2 = 22$, $M = 13$: do plots (i), (ii), and (iii)

Page 2: $a_2 = 1$, $D_2 = 21$, $M = 13$: do plots (i), (ii), and (iii)

Page 3: $a_2 = -1$, $D_2 = 21$, $M = 13$: do plots (i), (ii), and (iii)

Page 4: $a_2 = 1$, $D_2 = 22$, $M = 15$: do plots (i), (ii), and (iii)

Page 5: $a_2 = 1$, $D_2 = 21$, $M = 15$: do plots (i), (ii), and (iii)

Page 6: $a_2 = -1$, $D_2 = 21$, $M = 15$: do plots (i), (ii), and (iii)

Page 7: $a_2 = 1$, $D_2 = 22$, $M = 127$: do plots (i), (ii), and (iii)

Page 8: $a_2 = 1$, $D_2 = 21$, $M = 127$: do plots (i), (ii), and (iii)

Page 9: $a_2 = -1$, $D_2 = 21$, $M = 127$: do plots (i), (ii), and (iii)

Note 1: This homework is worth 15/3=5 points of your final grade.

Note 2: The goal of this Matlab homework is to exercise you on the practical applications of discrete-time cross-correlation. An additional goal is to get you started on using Matlab.

General Information.

Deliverables for this project include 27 plots. Each plot should be clearly labeled, and should be accompanied by a brief explanation. The collection of plots and accompanying explanations should be put together in a cohesive manner in the form of a very brief report. Don't go overboard – this is simply a homework, **not** a project. Append source code to the report.

You may use any Matlab command you like in solving these problems. Each student is expected to do his/her own work and each must turn in his/her own report. Again, your write-up for this homework should be in the form of a very brief report. Handwriting is acceptable but please be sure it is legible. Your report should include:

- Answers to all questions posed including mathematical development where necessary.
- The 27 plots and observations/explanations

2.62 Determine the autocorrelation sequences of the following signals.

(a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$

(b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

What is your conclusion?

2.63 What is the normalized autocorrelation sequence of the signal $x(n)$ given by

$$x(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$$

2.64 An audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling, is

$$x(n) = s(n) + r_1 s(n - k_1) + r_2 s(n - k_2)$$

where k_1 and k_2 are the delays of the two echoes.

(a) Determine the autocorrelation $r_{xx}(l)$ of the signal $x(n)$.

(b) Can we obtain r_1, r_2, k_1, and k_2 by observing $r_{xx}(l)$?

(c) What happens if $r_2 = 0$?

2.65 *Time-delay estimation in radar* Let $x_a(t)$ be the transmitted signal and $y_a(t)$ be the received signal in a radar system, where

$$y_a(t) = a x_a(t - t_d) + v_a(t)$$

and $v_a(t)$ is additive random noise. The signals $x_a(t)$ and $y_a(t)$ are sampled in the receiver, according to the sampling theorem, and are processed digitally to determine the time delay and hence the distance of the object. The resulting discrete-time signals are

$$\begin{aligned} x(n) &= x_a(nT) \\ y(n) &= y_a(nT) = a x_a(nT - DT) + v_a(nT) \\ &\triangleq a x(n - D) + v(n) \end{aligned}$$

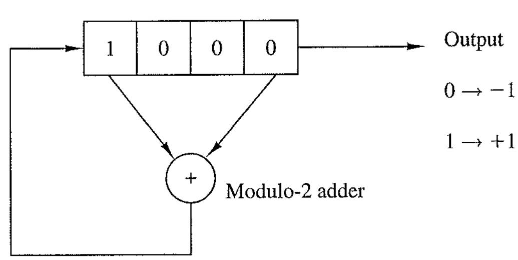

Figure P2.65
Linear feedback shift register.

(a) Explain how we can measure the delay D by computing the crosscorrelation $r_{xy}(l)$.

(b) Let $x(n)$ be the 13-point *Barker sequence*

$$x(n) = \{+1, +1, +1, +1, +1, -1, -1, +1, +1, -1, +1, -1, +1\}$$

and $v(n)$ be a Gaussian random sequence with zero mean and variance $\sigma^2 = 0.01$. Write a program that generates the sequence $y(n)$, $0 \le n \le 199$ for $a = 0.9$ and $D = 20$. Plot the signals $x(n)$, $y(n)$, $0 \le n \le 199$.

(c) Compute and plot the crosscorrelation $r_{xy}(l)$, $0 \le l \le 59$. Use the plot to estimate the value of the delay D.

(d) Repeat parts (b) and (c) for $\sigma^2 = 0.1$ and $\sigma^2 = 1$.

(e) Repeat parts (b) and (c) for the signal sequence

$$x(n) = \{-1, -1, -1, +1, +1, +1, +1, -1, +1, -1, +1, +1, -1, -1, +1\}$$

which is obtained from the four-stage feedback shift register shown in Fig. P2.65. Note that $x(n)$ is just one period of the periodic sequence obtained from the feedback shift register.

(f) Repeat parts (b) and (c) for a sequence of period $N = 2^7 - 1$, which is obtained from a seven-stage feedback shift register. Table 2.2 gives the stages connected to the modulo-2 adder for (maximal-length) shift-register sequences of length $N = 2^m - 1$.

TABLE 2.2 Shift-Register Connections for Generating Maximal-Length Sequences

m	Stages Connected to Modulo-2 Adder
1	1
2	1, 2
3	1, 3
4	1, 4
5	1, 4
6	1, 6
7	1, 7
8	1, 5, 6, 7
9	1, 6
10	1, 8
11	1, 10
12	1, 7, 9, 12
13	1, 10, 11, 13
14	1, 5, 9, 14
15	1, 15
16	1, 5, 14, 16
17	1, 15