# ECE 538 Digital Signal Processing I Final Exam 2018 Test Date: Monday, 10 December 2018 

## Cover Sheet

Test Duration: 120 minutes.
Open Book but Closed Notes.
Three $8.5 \times 11$ handwritten crib sheets allowed.
Calculators NOT allowed.
This test contains FIVE problems.
All work should be done in the proper space provided.

Good luck in this final exam!
Great having you in class this semester!
Have a joyous holiday season!

Problem 1. An OFDM signal is synthesized as a sum of four sinewaves of length 4 as

$$
\tilde{x}[n]=\frac{1}{4} \sum_{k=0}^{3} b_{k} e^{j 2 \pi \frac{k}{4} n}\{u[n]-u[n-4]\}
$$

Each of the four values $b_{k}$ is one of the four values listed in the symbol alphabet below.

$$
b_{k} \in\{1,3,5,7\} \quad k=0,1,2,3
$$

We know in advance that the signal we transmit will be convolved with a filter of length $L=3$. Thus, we add a cyclic prefix of length 3 which effective creates a sum of sinewaves of length 7 as prescribed below:

$$
x[n]=\frac{1}{4} \sum_{k=0}^{3} b_{k} e^{j 2 \pi \frac{k}{4} n}\{u[n+3]-u[n-4]\}
$$

The sequence of length 7 above is convolved with the filter below of length 3

$$
h[n]=\{4,2,1\}=4 \delta[n]+2 \delta[n-1]+1 \delta[n-2]
$$

This ultimately yields the following sequence of length $7+3-1=9$
$y[n]=x[n] * h[n]=\{-4-4 j,-6-2 j,-7+3 j, 13+2 j, 3-3 j,-2-2 j,-7+3 j,-3+2 j,-1+j\}$
Your task is to determine the numerical values of each of the four symbols: $b_{0}, b_{1}, b_{2}$, and $b_{3}$. Explain and show all the steps in determining your answers. Please lay your work out nicely with logical ordering. Your work will be more important than your final answers.

$$
\text { See solution to Prob } 1 \text { from Exam 1, Fall } 2017
$$

Answers:
$b 0=1, \quad b 1=3, \quad b 2=5, \quad b 3=7$

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## Problem 2.

(a) Determine the autocorrelation $r_{x_{0} x_{0}}[\ell]$ of the length- 4 sequence $x_{0}[n]$ below, which is written two different ways. Write your answer in sequence form indicating where the value for $\ell=0$ is located.

$$
x_{0}[n]=u[n]-u[n-4]=\{1,1,1,1\}
$$

(b) Determine the autocorrelation $r_{x_{1} x_{1}}[\ell]$ of the length- 4 sequence $x_{1}[n]$ below. Write your answer in sequence form indicating where the value for $\ell=0$ is located.

$$
x_{1}[n]=e^{j\left(\frac{\pi}{2} n+\frac{\pi}{\sqrt{2}}\right)}\{u[n]-u[n-4]\}
$$

(c) Determine the autocorrelation $r_{x_{2} x_{2}}[\ell]$ of the length- 4 sequence $x_{2}[n]$ below. Write your answer in sequence form indicating where the value for $\ell=0$ is located.

$$
x_{2}[n]=e^{j \pi(n-2)}\{u[n-2]-u[n-6]\}
$$

(d) Determine the autocorrelation $r_{x_{3} x_{3}}[\ell]$ of the length- 4 sequence $x_{3}[n]$ below. Write your answer in sequence form indicating where the value for $\ell=0$ is located.

$$
x_{3}[n]=e^{-j\left(\frac{\pi}{2} n+\frac{\pi}{\sqrt{2}}\right)}\{u[n]-u[n-4]\}
$$

(e) Sum your answers to parts (a) thru (d) to form the sum below. Do a stem plot of $r_{x x}[\ell]$.

$$
r_{x x}[\ell]=r_{x_{0} x_{0}}[\ell]+r_{x_{1} x_{1}}[\ell]+r_{x_{2} x_{2}}[\ell]+r_{x_{3} x_{3}}[\ell]
$$

[^0]Problem 3. Let $x[n]$ be of length $L=8$, i.e., $x[n]=0$ for $n<0$ and $n \geq 8$ and $h[n]$ be of length $M=5$, i.e., $h[n]=0$ for $n<0$ and $n \geq 5$. Let $X_{8}(k)$ and $H_{8}(k)$ denote 8 -point DFT's of $x[n]$ and $\mathrm{h}[\mathrm{n}]$, respectively. The 8 -point inverse DFT of the product $Y_{8}(k)=$ $X_{8}(k) H_{8}(k)$, denoted $y_{8}[n]$, produces the following values:

| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{8}[n]$ | 75 | 58 | 49 | 48 | 55 | 70 | 85 | 100 |

Let $X_{10}(k)$ and $H_{10}(k)$ denote the 10-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 10 -point inverse DFT of the product $Y_{10}(k)=X_{10}(k) H_{10}(k)$, denoted $y_{10}[n]$, produces the following values:

| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{10}[n]$ | 28 | 22 | 26 | 40 | 55 | 70 | 85 | 100 | 70 | 44 |

Given $y_{8}[n]$ and $y_{10}[n]$, find the linear convolution of $x[n]$ and $h[n]$, i. e., list all of the numerical values of $y[n]=x[n] * h[n]$. Hint: all the values are positive, integers.

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$x[n]: L=8 \quad h[n]: M=5$
$y[n]=x[n] * h[n]:$ length $8+5-1=12$
$y_{10}[n] \Rightarrow y_{10}[n]=y[n]+y[n+10]\left\{\begin{array}{l}12-10=2 \\ \text { pts. aliased }\end{array}\right.$
$\left.y_{8}[n] \Rightarrow y_{8}[n]=y[n]+y[n+2]\right\} \begin{aligned} & 12-8=4 \\ & p+s . \text { al inced }\end{aligned}$

$$
\Rightarrow y[n]=y_{10}[n], n=2,3, \ldots, 9
$$

(A)

$$
\left.\begin{array}{c}
y_{8}[0]=y[0]+y[8] \\
75=y[0]+70
\end{array}\right\} y[0]=5
$$

B

$$
\left.\begin{array}{rl}
y_{8}[1] & =y[1]+y[9] \\
58 & =y[1]+44
\end{array}\right\} y[1]=14
$$

(C)

$$
\left.\begin{array}{rl}
y_{8}[2] & =y[2]+y[10] \\
49 & =26+y[10]
\end{array}\right\} y[10]=23
$$

D

$$
\left.\begin{array}{rl}
49 & =26+y[10] \\
y_{8}[3] & =y[3]+y[1] \\
48 & =40+y[11]
\end{array}\right\}
$$

Check:

$$
\begin{aligned}
y[n] & y[1]=\left\{5,14,26,40,55,70,85,10070, x^{23}, 8\right\} \\
& =\{5,14,26,40,55,70,85,100,70,7044,23,8\}
\end{aligned}
$$

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Problem 4. (a)
(a) Consider $h_{1}[n]$ and $h_{2}[n]$ to be two distinct all-pass filters $\left(p_{1} \neq p_{2}\right)$ with respective impulse responses below; $p_{1}$ and $p_{2}$ are both real-valued with absolute value less than 1. Is the sum $h[n]=h_{1}[n]+h_{2}[n]$ an all-pass filter for any and all values of $p_{1}$ and $p_{2}$ $\left(p_{1} \neq p_{2}\right)$ ?? Explain your answer. Your explanation is much more important than your answer.

$$
\begin{align*}
& h_{1}[n]=\frac{1}{p_{1}}\left\{\delta[n]+\left(p_{1}^{2}-1\right) p_{1}^{n} u[n]\right\}  \tag{1}\\
& h_{2}[n]=\frac{1}{p_{2}}\left\{\delta[n]+\left(p_{2}^{2}-1\right) p_{2}^{n} u[n]\right\} \tag{2}
\end{align*}
$$

See solution to Prob 2(a) from Exam 1, Fall 2015

Problem 4. (b)
(b) Consider $h_{1}[n]$ and $h_{2}[n]$ to be two distinct all-pass filters $\left(p_{1} \neq p_{2}\right)$ with respective impulse responses below; $p_{1}$ and $p_{2}$ are both real-valued with absolute value less than 1. Is the convolution $h[n]=h_{1}[n] * h_{2}[n]$ an all-pass filter for any and all values of $p_{1}$ and $p_{2}$ ? Explain your answer. Your explanation is much more important than your answer.

$$
\begin{align*}
& h_{1}[n]=\frac{1}{p_{1}}\left\{\delta[n]+\left(p_{1}^{2}-1\right) p_{1}^{n} u[n]\right\}  \tag{3}\\
& h_{2}[n]=\frac{1}{p_{2}}\left\{\delta[n]+\left(p_{2}^{2}-1\right) p_{2}^{n} u[n]\right\} \tag{4}
\end{align*}
$$

See solution to Prob 1 from Exam 1, Fall 2015

## Problem 4.(c)

(c) Consider $h[n]$ to be an all-pass filter with respective impulse response below.

$$
\begin{equation*}
h[n]=\frac{1}{p}\left\{\delta[n]+\left(p^{2}-1\right) p^{n} u[n]\right\} \tag{5}
\end{equation*}
$$

Is the product

$$
g[n]=e^{j \omega_{o} n} h[n]
$$

an all-pass filter for any and all values of the frequency $\omega_{o}$ ? Explain your answer. Your explanation is much more important than your answer.

See solution to Prob 2(b) from Exam 1, Fall 2015

## Problem 5.

(a) Consider sampling the DTFT below at $N=8$ equi-spaced frequencies over $0 \leq \omega<2 \pi$.

$$
X(\omega)=\frac{1}{1-0.8 e^{-j \omega}}+\frac{1}{1-0.9 e^{-j \omega}}
$$

That is, let $X_{8}(k)=X(2 \pi k / 8), k=0,1, \ldots 7$, where $X(\omega)$ is defined above. Now, compute $x_{8}[n]$ as the 8 -point Inverse DFT of $X_{8}(k)$. Determine a closed-form ("plug-and-chug") expression for $x_{8}[n]$ that prescribes the value of $x_{8}[n]$ for $n=0,1, \ldots, 7$. Show all work. Note: A closed-form expression contains NO summations and it is NOT a listing of numbers. Hint:

$$
\frac{1}{1-(.9)^{8}}=1.7558 \quad \frac{1}{1-(.8)^{8}}=1.2016
$$

(b) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined as below.

$$
X(\omega)=\frac{8}{1-0.8 e^{-j \omega}}-\frac{9}{1-0.9 e^{-j \omega}}
$$

(c) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined as below.

$$
X(\omega)=\frac{e^{j 2 \omega}}{1-0.8 e^{-j \omega}}+\frac{e^{j 4 \omega}}{1-0.9 e^{-j \omega}}
$$

(d) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined below. (That is, the two terms are now multiplied rather than being summed as in part (a).) Show all work.

$$
X(\omega)=\frac{1}{1-0.8 e^{-j \omega}} \frac{1}{1-0.9 e^{-j \omega}}
$$

See solution to Prob 4 from Final Exam, Fall 2015

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[^0]:    See solution to Prob 2 from Exam 1, Fall 2014

