NAME: **2019**
Digital Signal Processing I **Final Exam** **Fall 2019**
Thursday, PHYS 203 **12 Dec. 2019**

Cover Sheet

Test Duration: 120 minutes.
Open Book but Closed Notes. Three 8.5 x 11 crib sheets allowed
Calculators NOT allowed.
This test contains **FOUR** problems.
All work should be done on the blank pages provided.
Your answer to each part of the exam should be clearly labeled.

Problem 4.

(a) Consider sampling the DTFT below at $N = 8$ equi-spaced frequencies over $0 \leq \omega < 2\pi$.

$$X(\omega) = \frac{1}{1 - 0.5e^{-j\omega}} + \frac{1}{1 - 0.75e^{-j\omega}}$$

That is, let $X_8(k) = X(2\pi k/8)$, $k = 0, 1, ...7$, where $X(\omega)$ is defined above. Now, compute $x_8[n]$ as the 8-point Inverse DFT of $X_8(k)$. Determine a closed-form ("plug-and-chug") expression for $x_8[n]$ that prescribes the value of $x_8[n]$ for $n = 0, 1, ..., 7$. Show all work. **Note:** A *closed-form* expression contains NO summations and it is NOT a listing of numbers. *Hint:*

$$c_1 = \frac{1}{1 - (.5)^8} = 1.0039 \qquad c_2 = \frac{1}{1 - (.75)^8} = 1.1113$$

(b) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined as below.

$$X(\omega) = \frac{3}{1 - 0.5e^{-j\omega}} - \frac{2}{1 - 0.75e^{-j\omega}}$$

(c) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined as below.

$$X(\omega) = \frac{e^{j2\omega}}{1 - 0.5e^{-j\omega}} + \frac{e^{j4\omega}}{1 - 0.75e^{-j\omega}}$$

(d) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined as below.

$$X(\omega) = \frac{1}{1 - 0.5e^{-j(\omega - 2\pi/8)}} + \frac{1}{1 - 0.75e^{-j(\omega - 6\pi/8)}}$$

(e) Show how your answer changes when everything is the same as defined in part (a) except that the DTFT is now defined below. (That is, the two terms are now multiplied rather than being summed as in part (a).) Show all work.

$$X(\omega) = \frac{1}{1 - 0.5e^{-j\omega}} \frac{1}{1 - 0.75e^{-j\omega}}$$

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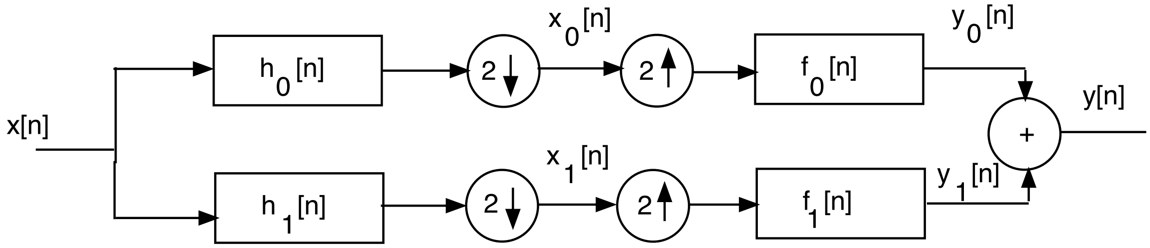
Problem 2. In the system below, the two analysis filters, $h_0[n]$ and $h_1[n]$, and the two synthesis filters, $f_0[n]$ and $f_1[n]$, form a Quadrature Mirror Filter (QMF). Specifically,

$$h_1[n] = e^{j\pi n} h_0[n] \qquad f_0[n] = h_0[n] \qquad f_1[n] = -h_1[n]$$

The lowpass filter $h_0[n]$ employed is the following filter:

$$h_0[n] = \left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right) \frac{\sin\left[\frac{\pi}{3}\left(n+\frac{1}{2}\right)\right]}{\pi\left(n+\frac{1}{2}\right)} + \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) \frac{\sin\left[\frac{2\pi}{3}\left(n+\frac{1}{2}\right)\right]}{\pi\left(n+\frac{1}{2}\right)} + \frac{1}{2}\frac{\sin\left[\pi\left(n+\frac{1}{2}\right)\right]}{\pi\left(n+\frac{1}{2}\right)} \quad (1)$$



Part I Determine mathematically/graphically if the lowpass filter above satisfies the condition required for Perfect Reconstruction. Clearly state what that condition is (don't need to re-derive it) and then show whether it is satisfied with the filter $h_0[n]$, showing as much detail as possible. **VIP:** for the sake of time, you can (correctly) assume that the net effect of the shift to the left by 0.5 in (discrete) time causes multiplication by the term $e^{j0.5\omega}$ in the frequency domain.

Part II For Part II of this problem, the input to the QMF is the sum of three infinite-length sinewaves as defined below. There are 3 frequencies present in $x[n]$: $\frac{3\pi}{16}, \frac{7\pi}{16}, \frac{15\pi}{16}$

$$x[n] = \cos\left(\frac{3\pi}{16}n\right) + \cos\left(\frac{7\pi}{16}n\right) + \cos\left(\frac{15\pi}{16}n\right)$$

(a) Determine and list the frequencies present in $x_0[n]$ in the range $0 \leq \omega \leq \pi$.

(b) Determine and list the frequencies present in $x_1[n]$ in the range $0 \leq \omega \leq \pi$.

(c) Determine and list the frequencies present in $y_0[n]$. Show and explain your work. Plot the DTFT of $y_0[n]$, $Y_0(\omega)$. *Hint:* There are more than 3 frequencies present in $y_0[n]$.

(d) Determine and list the frequencies present in $y_1[n]$. Show and explain your work. Plot the DTFT of $y_1[n]$, $Y_1(\omega)$. *Hint:* There are more than 3 frequencies present in $y_1[n]$.

Part III

(a) Draw a block diagram of the computationally efficient implementation of the analysis side of the Two-Channel QMF in the figure above.

(b) Plot the magnitude of the DTFT (frequency response) of each of the two polyphase components, $h_0[2n]$ and $h_0[2n+1]$, of the lowpass filter $h_0[n]$ defined in Eqn (1.)

(c) Draw a block diagram of the computationally efficient implementation of the synthesis side of the Two-Channel QMF in the figure above.

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BE SURE TO CLEARLY LABEL YOUR ANSWER TO EACH PART .

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Problem 3.
A second-order digital filter is to be designed from an analog filter having two poles in the s-plane at $-0.2 + 0.4j$ and $-0.2 - 0.4j$ and two zeros at $j\sqrt{3}$ and $-j\sqrt{3}$, via the bilinear transformation method characterized by the mapping

$$s = \frac{z-1}{z+1}$$

Note that $-0.2 + 0.4j = -(1/5) + j(2/5)$ and $j\sqrt{3} = j\tan(\pi/3)$

(a) Is the resulting digital filter (BIBO) stable? Briefly explain why or why not.

(b) Denote the frequency response of the resulting digital filter as $H(\omega)$ (the DTFT of its impulse response). You are given that in the range $0 < \omega < \pi$, there is only one value of ω for which $H(\omega) = 0$. Determine that value of ω.

(c) Draw a pole-zero diagram for the resulting **digital** filter. Give the exact locations of the poles and zeros of the digital filter in the z-plane.

(d) Plot the magnitude of the DTFT of the resulting digital filter, $|H(\omega)|$, over $-\pi < \omega < \pi$. You are given that $H(0) = 6$. Be sure to indicate any frequency for which $|H(\omega)| = 0$. Also, specifically note the numerical value of $|H(\omega)|$ for $\omega = \frac{\pi}{2}$ and $\omega = \pi$.

(e) Again, given that $H(0) = 6$, determine the difference equation for the resulting digital filter.

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Problem 4.

(a) Consider a causal FIR filter of length $M = 14$ with impulse response

$$h[n] = u[n] - u[n-14]$$

Provide a **closed-form** expression for the 16-pt DFT of $h[n]$, denoted $H_{16}(k)$, as a function of k. Simplify as much as possible. You are given the following four values:

$$H_{16}(0) = 14 \quad H_{16}(4) = 1 - j = \sqrt{2}e^{-j\frac{\pi}{4}} \quad H_{16}(8) = 0 \quad H_{16}(12) = 1 + j = \sqrt{2}e^{j\frac{\pi}{4}}$$

(b) Consider the sequence $x[n]$ of length $L = 16$ below, equal to a sum of several finite-length sinewaves.

$$x[n] = 1 + 2\cos\left(\frac{\pi}{2}n\right) + \cos(\pi n), \quad n = 0, 1, ..., 15.$$

$y_{16}[n]$ is formed by computing $X_{16}(k)$ as a 16-pt DFT of $x[n]$, $H_{16}(k)$ as a 16-pt DFT of $h[n]$, and then $y_{16}[n]$ as the 16-pt inverse DFT of $Y_{16}(k) = X_{16}(k)H_{16}(k)$. Express the result $y_{16}[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

(c) For the remaining parts of this problem, $h[n]$ is now defined as the causal FIR filter of length $M = 14$ below.

$$h[n] = (-1)^n\{u[n] - u[n-14]\}$$

Provide a **closed-form** expression for the 16-pt DFT of $h[n]$, denoted $H_{16}(k)$, as a function of k. Simplify as much as possible.

(d) Consider again the sequence $x[n]$ of length $L = 16$ below.

$$x[n] = 1 + 2\cos\left(\frac{\pi}{2}n\right) + \cos(\pi n), \quad n = 0, 1, ..., 15.$$

$y_{16}[n]$ is formed by computing $X_{16}(k)$ as an 16-pt DFT of $x[n]$, $H_{16}(k)$ as a 16-pt DFT of $h[n]$, and then $y_{16}[n]$ as the 16-pt inverse DFT of $Y_{16}(k) = X_{16}(k)H_{16}(k)$. Express the result $y_{16}[n]$ as a weighted sum of finite-length sinewaves.

(e) Consider the sequence $p[n]$ of length $L = 16$ below.

$$p[n] = 4\delta[n] + 4\delta[n-4] + 4\delta[n-8] + 4\delta[n-12]$$

Is $p[n] = x[n]$? If they are equal, provide an explanation as to why they are equal.

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