NAME: **Final Exam**
ECE 538 Digital Signal Processing I **13 Dec. 2012**
Room: ARMS B061 **3:30-5:30 pm**

Cover Sheet

WRITE YOUR NAME ON THIS COVER SHEET

Test Duration: 120 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
All work should be done in the space provided.

Problem 1. (a) Consider a signal, $x[n]$, of length $L = 7$ and an FIR filter with impulse response $h[n]$ of length $M = 2$ as described below.

$$\begin{aligned} h[n] &= \{1, -1\} \\ x[n] &= \{1, 2, 3, 4, 3, 2, 1\} \end{aligned}$$

We compute an $N = 9$-pt. DFT of each of these two sequences as

$$x[n] \underset{9}{\overset{\text{DFT}}{\longleftrightarrow}} X_9[k] \qquad\qquad h[n] \underset{9}{\overset{\text{DFT}}{\longleftrightarrow}} H_9[k]$$

Next, we point-wise multiply to form $Y_9[k] = X_9[k]H_9[k]$, $k = 0, 1, ..., 8$. Finally, we compute an $N = 9$-pt. inverse DFT of $Y_9[k]$ to obtain $y_9[n]$. Determine the numerical values of $y_9[n]$ for $n = 0, 1, ..., 8$. You can write out $y_9[n]$ in either sequence form OR do a stem plot.

Problem 1. Part (b): Same $x[n]$ and $h[n]$ as described below:

$$\begin{aligned} h[n] &= \{1, -1\} \\ x[n] &= \{1, 2, 3, 4, 3, 2, 1\} \end{aligned}$$

We evaluate the DTFT of $x[n]$ at the $N = 4$ equi-spaced frequencies $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.; the same for $h[n]$.

$$x[n] \overset{\text{DFT}}{\underset{4}{\longleftrightarrow}} X_4[k] \qquad\qquad h[n] \overset{\text{DFT}}{\underset{4}{\longleftrightarrow}} H_4[k]$$

Next, we point-wise multiply to form $Y_4[k] = X_4[k]H_4[k]$, $k = 0, 1, 2, 3$. Finally, we compute an $N = 4$-pt. inverse DFT of $Y_4[k]$ to obtain $y_4[n]$. Determine the numerical values of $y_4[n]$ for $n = 0, 1, 2, 3$. You can write out $y_4[n]$ in either sequence form OR do a stem plot.

Problem 2. As part of the first stage in a radix 2 FFT, a sequence $x[n]$ of length $N = 8$ is decomposed into two sequences of length 4 as

$$\begin{aligned} f_0[n] &= x[2n], \quad n = 0, 1, 2, 3 \\ f_1[n] &= x[2n+1], \quad n = 0, 1, 2, 3 \end{aligned}$$

We compute a 4 -pt. DFT of each of these two sequences as

$$f_0[n] \underset{4}{\overset{\text{DFT}}{\longleftrightarrow}} F_0[k] \qquad\qquad f_1[n] \underset{4}{\overset{\text{DFT}}{\longleftrightarrow}} F_1[k]$$

The specific values of $F_0[k]$ and $F_1[k]$, $k = 0, 1, 2, 3$, obtained from the length $N = 8$ sequence in question are listed in the Table below.

k	0	1	2	3
$F_0[k]$	0	0	1	1
$F_1[k]$	0	0	-j	$\frac{1}{\sqrt{2}}(-1+j)$
W_8^k	1	$\frac{1}{\sqrt{2}}(1-j)$	-j	$-\frac{1}{\sqrt{2}}(1+j)$

(a) From the values of $F_0[k]$ and $F_1[k]$, $k = 0, 1, 2, 3$, and the values of $W_8^k = e^{-j\frac{2\pi}{8}k}$, $k = 0, 1, 2, 3$, provided in the Table, determine the numerical values of the actual $N = 8$-pt. DFT of $x[n]$ denoted $X_8[k]$. You can either write out the numerical values of $X_8[k]$ for $k = 0, 1, 2, 3, 4, 5, 6, 7$, in sequence form or do a stem plot.

(b) The underlying length $N = 8$ sequence $x[n]$ may be expressed as

$$x[n] = \frac{1}{4}e^{j2\pi\frac{k_1}{8}n} + \frac{1}{4}e^{j2\pi\frac{k_2}{8}n}, \quad n = 0, 1, ..., 7.$$

where k_1 and k_2 are both integers between 0 and 7, that is, $k_i \in \{0, 1, 2, 3, 4, 5, 6, 7\}$, $i = 1, 2$. Given the values of $X_8[k]$ for $k = 0, 1, ..., 7$, determined in part (a), determine the numerical values of k_1 and k_2 (each should be an integer between 0 and 7.)

Do all your work for both parts of this problem on the next blank page

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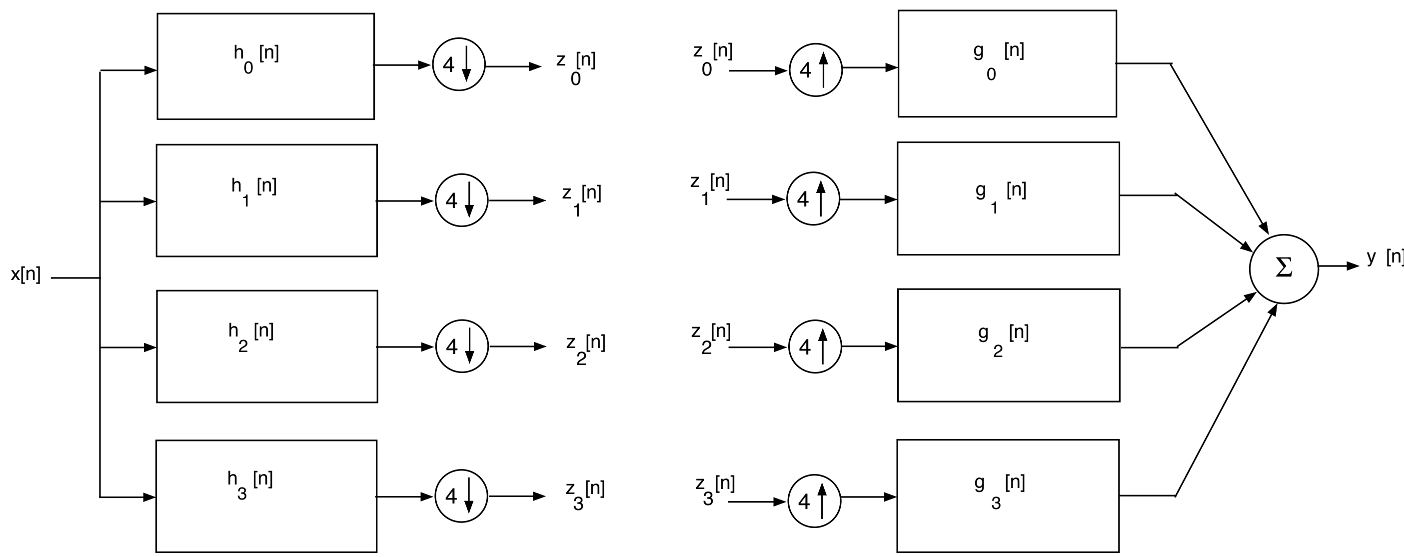

Figure 1(a). Analysis Filter Bank. Figure 1(b). Synthesis Filter Bank.

Figure 1. For Problem 3 on Next Page.

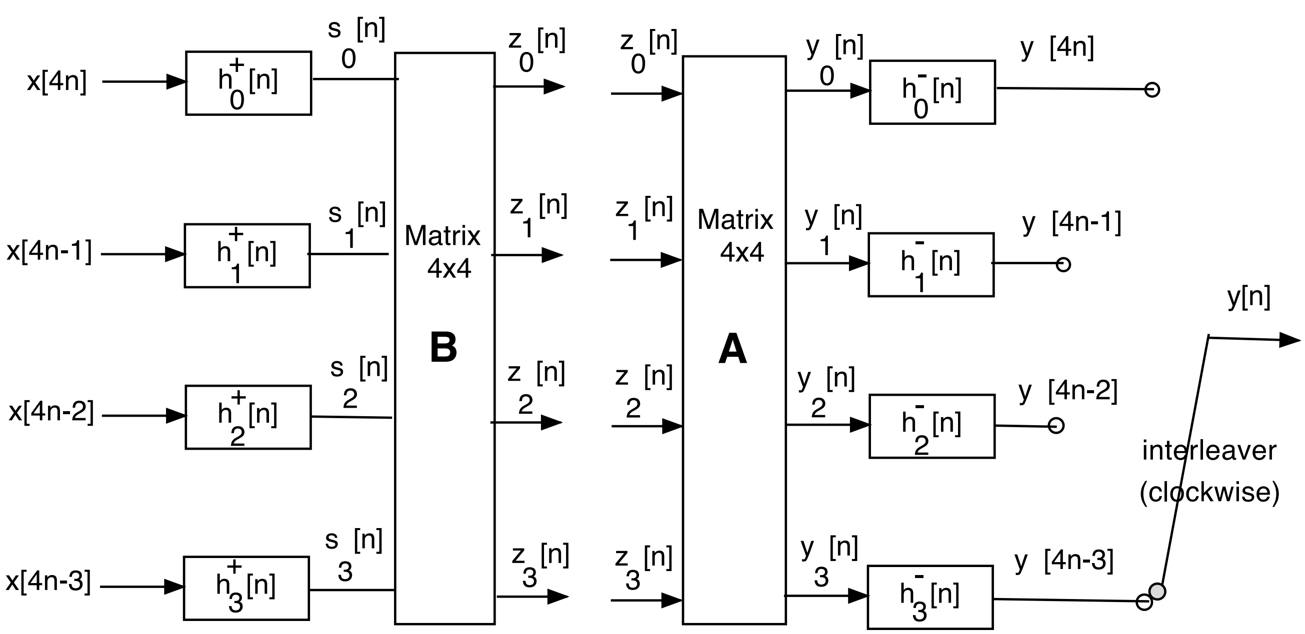

Figure 2. For Problem 3 on Next Page.

Problem 3. Consider the $M = 4$ channel Filter Bank in Figure 1 on the previous page. You are to determine whether it achieves Perfect Reconstruction with the following causal analysis filters, each of which is of length 4 and starts at $n = 0$.

$$h_0[n] = \{1, 1, 1, 1\}$$

$$h_1[n] = \{1, 1, -1, -1\}$$
$$h_2[n] = \{1, -1, 1, -1\}$$
$$h_3[n] = \{1, -1, -1, 1\}$$

The corresponding synthesis filters are defined below. The four analysis filters and the four synthesis filters are all real-valued.

$$g_k[n] = h_k[-n], \quad k = 0, 1, 2, 3$$

In the space provided on the next two blank pages, you need to derive and clearly specify all filters and matrices in the computationally efficient implementation of this filter bank drawn in Figure 2 on the previous page. All answers are real-valued quantities.

NOTE: 1 The matrices **A** and **B** are **real-valued** matrices that have nothing to do with a 4-pt DFT matrix.

NOTE 2: After clearly specifying all the filters and matrices in Figure 2, determine if the Filter Bank achieves Perfect Reconstruction. Explain your answer.

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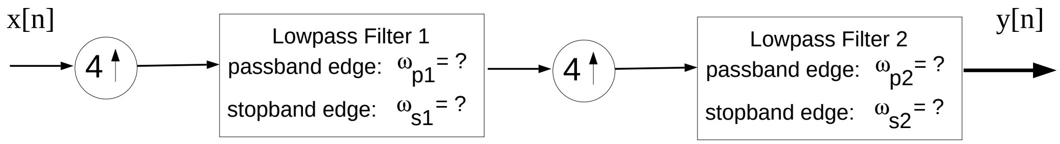
Problem 4. Part (a). The signal $x[n] = x_a(n/F_s)$ is obtained by sampling an analog signal $x_a(t)$ having a bandwidth of W at a rate of $F_s = \frac{8}{3}W$. The sampling rate is increased by a factor of $L = 16$ in two stages via the system below.



(i) Determine the passband edge, ω_{p1}, and stopband edge, ω_{s1}, of the first lowpass filter.

(ii) Determine the passband edge, ω_{p2}, and stopband edge, ω_{s2}, of the second lowpass filter. Show all work below.

Problem 4. Part (b). Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude of the DTFT of $h[n]$ over $-\pi < \omega < \pi$.

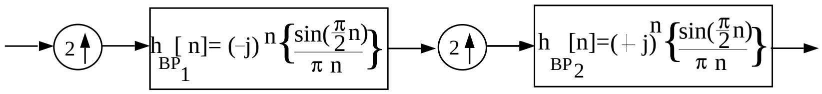


Figure 3(a).

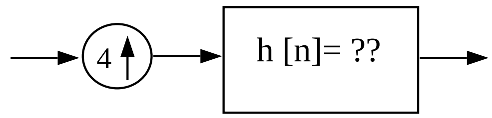


Figure 3(b).

Problem 4. Part (c). Determine the impulse response $h_{\text{eq}}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). Simplify your answer as much as possible for full credit.

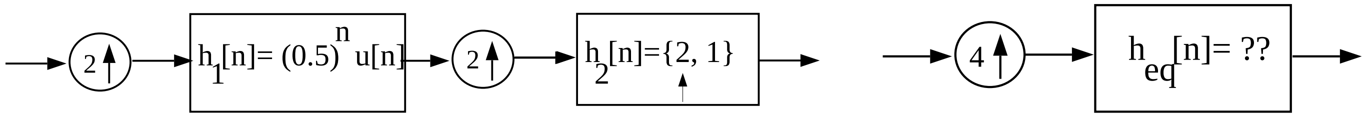


Figure 4(a). Figure 4(b).

Problem 5. Do all work for this problem on the next few blank pages. Clearly delineate your work and answer for each part.

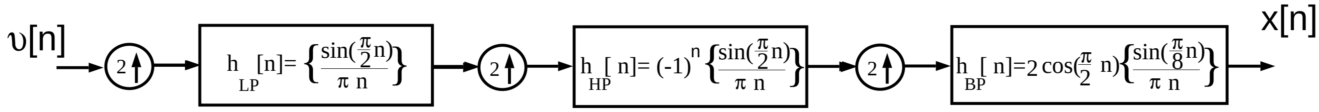


Figure 5(a).

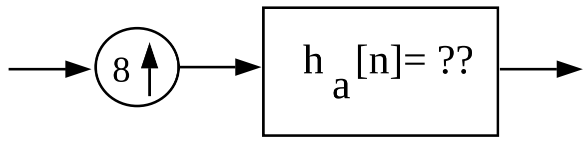


Figure 5(b).

(a) Determine the impulse response $h_a[n]$ in Figure 5(b) so that the I/O relationship of the system in Figure 5(b) is exactly the same as the I/O relationship of the system in Figure 5(a). Plot the magnitude of the DTFT of $h_a[n]$ over $-\pi < \omega < \pi$.

(b) Determine the autocorrelation sequence $r_{\nu\nu}[\ell]$ for the input $\nu[n]$ below with $p = \frac{3}{4}$.

$$\nu[n] = \frac{1}{p}\left\{\delta[n] \; + (p^2 - 1)p^n u[n]\right\} \tag{1}$$

(c) Determine and plot the DTFT of the **output** autocorrelation sequence $r_{xx}[\ell]$ over $-\pi \le \omega \le \pi$.

$$S_{xx}(\omega) = \sum_{\ell=-\infty}^{\infty} r_{xx}[\ell] e^{-j\omega\ell}$$

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