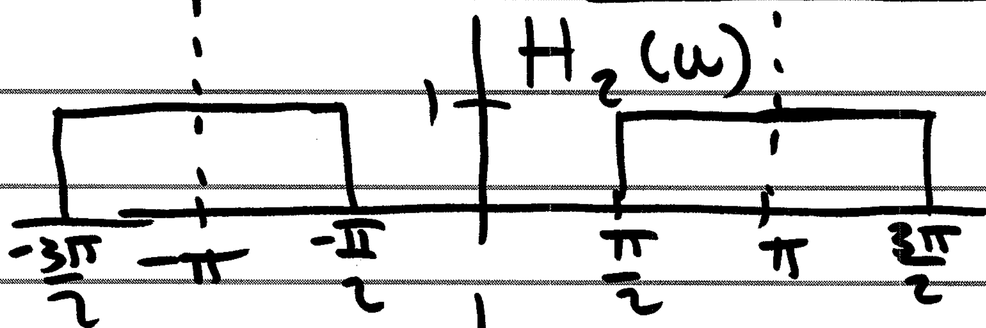
Prob. 2 F '05 FINAL EXAM

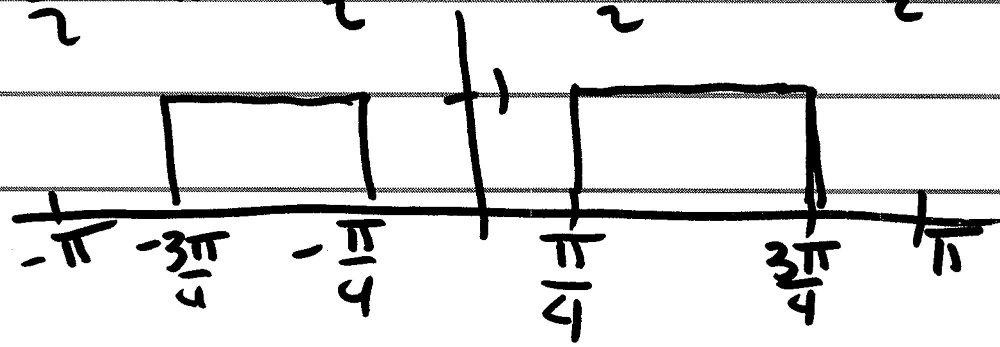
$$\rightarrow \boxed{2\uparrow} \rightarrow \boxed{H_1(\omega)} \rightarrow \boxed{2\uparrow} \rightarrow \boxed{H_2(\omega)} \rightarrow \boxed{2\uparrow} \rightarrow \boxed{H_3(\omega)} \rightarrow$$

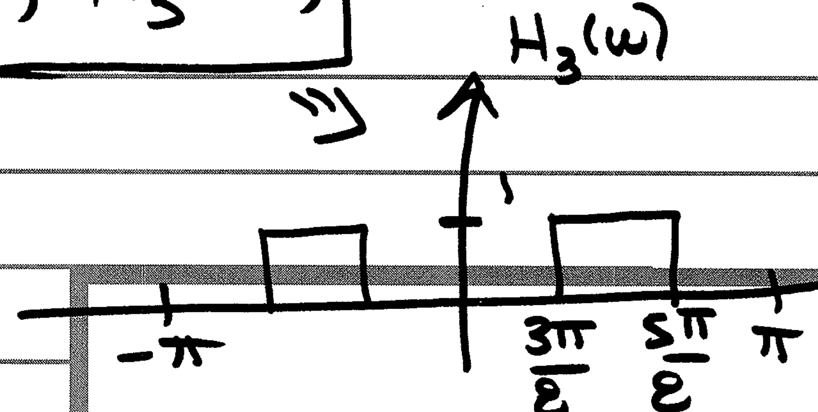
$$\rightarrow \boxed{2\uparrow} \rightarrow \boxed{H_1(\omega)} \rightarrow \boxed{4\uparrow} \rightarrow \boxed{H_2(2\omega)\, H_3(\omega)} \rightarrow$$

$$\rightarrow \boxed{8\uparrow} \rightarrow \boxed{H_1(4\omega)\, H_2(2\omega)\, H_3(\omega)}$$

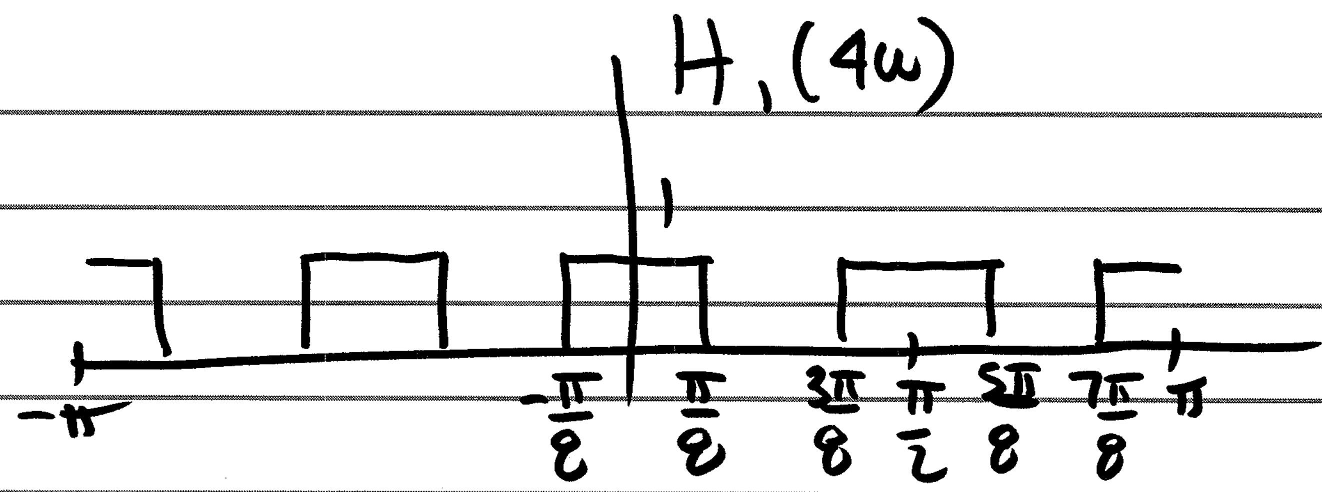
$\Rightarrow$







$$H_2(2\omega)\, H_3(\omega) = H_3(\omega)$$



$$H_1(4\omega)\,H_2(2\omega)\,H_3(\omega) = H_3(\omega)$$

$$S'_{xx}(\omega) = |H_a(\omega)|^2 \underbrace{S_{vv}(\omega)}_{\mathcal{F}\{\delta[m]\}}$$

$$= H_3(\omega)$$

$$= H_{BP}(\omega)$$

$$r_{xx}[m] = 2\cos\left(\frac{\pi}{2}m\right)\frac{\sin\left(\frac{\pi}{8}m\right)}{\pi m}$$

$$a_1(1) = -\frac{r_{xx}[1]}{r_{xx}[0]} = 0$$

$$\varepsilon_{min}^{(1)} = r_{xx}[0] = 1/4$$

$$2(-1)\frac{\frac{1}{\sqrt{2}}}{\pi 2}$$

$$\begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix}\begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = -\begin{bmatrix} 0 \\ -\frac{1}{\pi\sqrt{2}} \end{bmatrix}$$

$$a_2(1) = 0 \qquad a_2(2) = \frac{4}{\pi\sqrt{2}}$$

$$\varepsilon_{min}^{(2)} = \varepsilon_{min}^{(1)}\left(1 - a_2^2(2)\right) = \frac{1}{4}\left\{1 - \left(\frac{8}{\pi^2}\right)\right\}$$

Prob. 3 F'05 FINAL EXAM

$$h[n] = -a_1 h[n-1] - a_2 h[n-2]$$

$$- b_0 \delta[n] - b_1 \delta[n-1]$$

$$h[2] = -a_1 h[1] - a_2 h[0]$$

$$h[3] = -a_1 h[2] - a_2 h[1]$$

$$\begin{bmatrix} 13 & 35 \\ 5 & 13 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = - \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$p_1 = 1/2$

$$H(z) = \frac{-b_0 - b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$p_2 = 1/3$

Prob. 4 F' 05 FINAL EXAM

$$\begin{bmatrix} 2 & -j\sqrt{2} \\ j\sqrt{2} & 2 \end{bmatrix} \begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = - \begin{bmatrix} j\sqrt{2} \\ 0 \end{bmatrix}$$

$$z^2 + a_2(1) z + a_2(2)$$
$$= (z - e^{j\omega_1})(z - e^{j\omega_2})$$

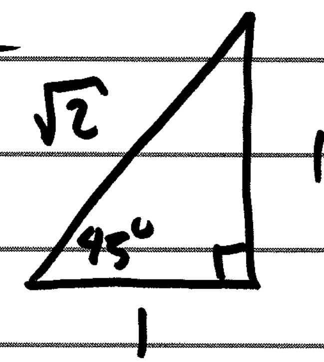
$$a_2(1) = \frac{\begin{vmatrix} -j\sqrt{2} & -j\sqrt{2} \\ 0 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -j\sqrt{2} \\ j\sqrt{2} & 2 \end{vmatrix}} = \frac{-j2\sqrt{2}}{4-2} = -j\sqrt{2}$$

$$j\sqrt{2}\left(-j\sqrt{2}\right) + 2a_2(2) = 0$$

$$a_2(2) = -1$$

$$z^2 - j\sqrt{2}\,z - 1 = 0$$

$$z = \frac{j\sqrt{2} \pm \sqrt{-2+4}}{2}$$



$$= \pm\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} =$$

$$+\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \qquad , \qquad -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$e^{j\frac{\pi}{4}} \qquad\qquad e^{j\frac{3\pi}{4}}$$

$$\omega_1 = \pi/4 \qquad\qquad \omega_2 = 3\pi/4$$

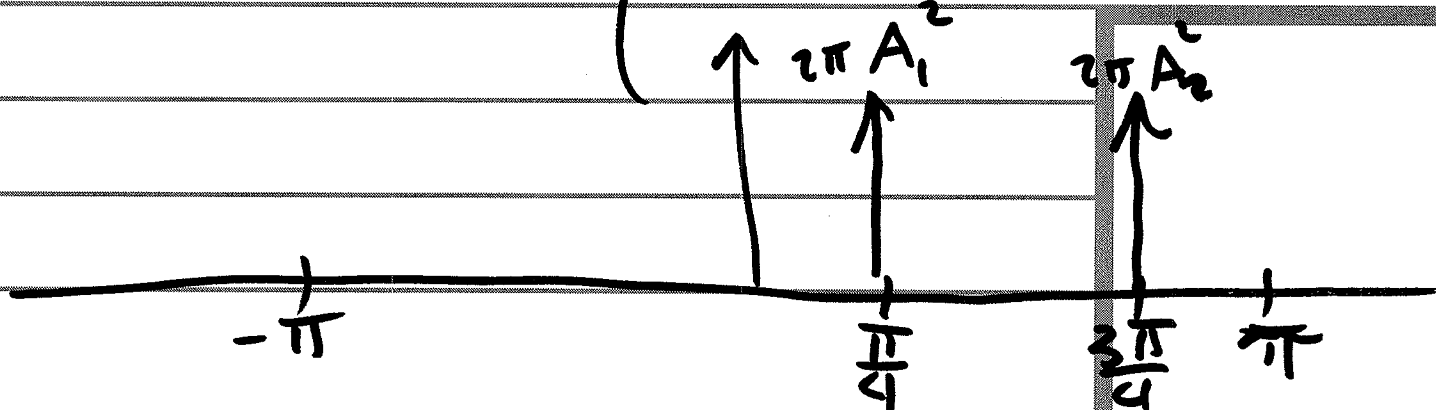
$$r_{xx}[3] = -a_2(1)\, r_{xx}[2] - a_2(2)\, r_{xx}[1]$$

$$= j\sqrt{2}\,(0) + 1\,(j\sqrt{2})$$

$$= j\sqrt{2}$$

$$S_{xx}(\omega) = \mathcal{F}\{r_{xx}[m]\} = ?$$

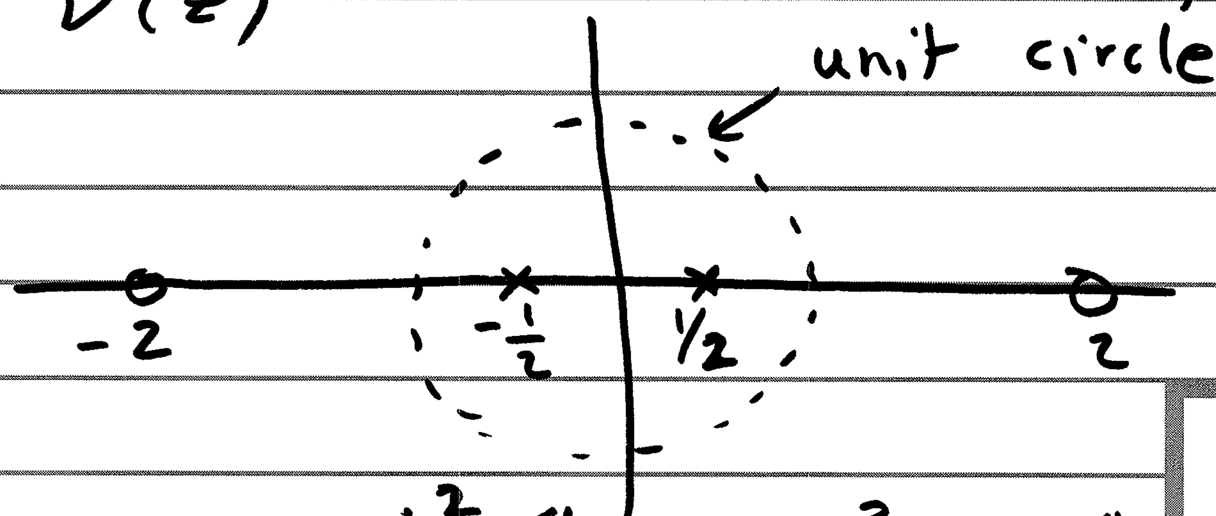
$$= \mathcal{F}\left\{A_1^2 e^{j\omega_1 m} + A_2^2 e^{j\omega_2 m}\right\}$$



Prob. 5 Final F'05

$$x[n] = \frac{1}{4} x[n-2] + v[n] - 4 v[n-2]$$

$$\frac{X(z)}{V(z)} = \frac{1 - 4z^{-2}}{1 - \frac{1}{4} z^{-2}} = \frac{z^2 - 4}{z^2 - \frac{1}{4}} = \frac{(z+2)(z-2)}{(z+\frac{1}{2})(z-\frac{1}{2})}$$



all-pass filter

$$H(0) = \frac{1^2 - 4}{1^2 - \frac{1}{4}} = \frac{-3}{\frac{3}{4}} = -4$$

$$|H(\omega)|^2 = 16 \quad \forall\ \omega$$

(b) $S_{xx}(\omega) = |H(\omega)|^2 \; S_{vv}(\omega)$

$= 16 \; (2)$

$= 32 \quad \forall \; \omega$

(a) $r_{xx}[m] = 32 \; \delta[m]$

(c) $a_1(1) = \, ? = 0 = \dfrac{-r_{xx}[1]}{r_{xx}[0]} =$

$\mathcal{E}_{min}^{(1)} = r_{xx}[0] = 32$

(d) $a_2(1) = 0, \; a_2(2) = 0$

$\mathcal{E}_{min}^{(2)} = 32$