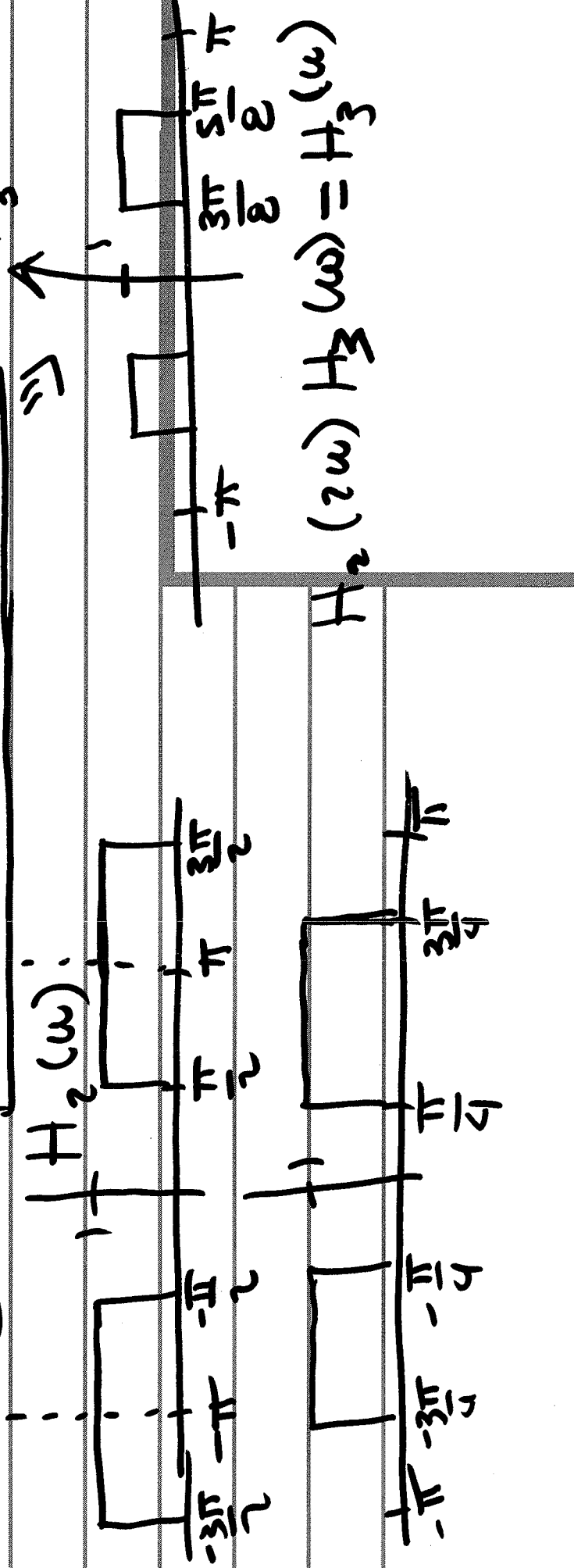
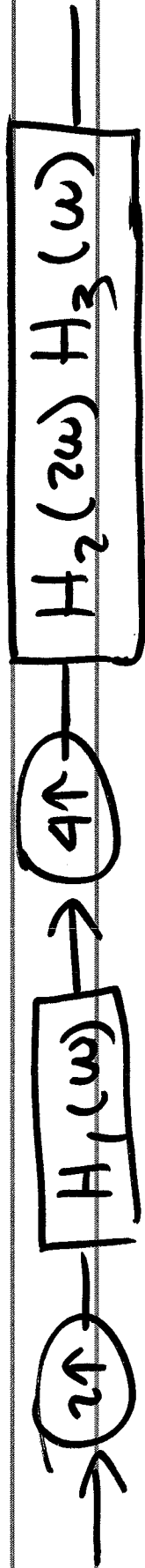
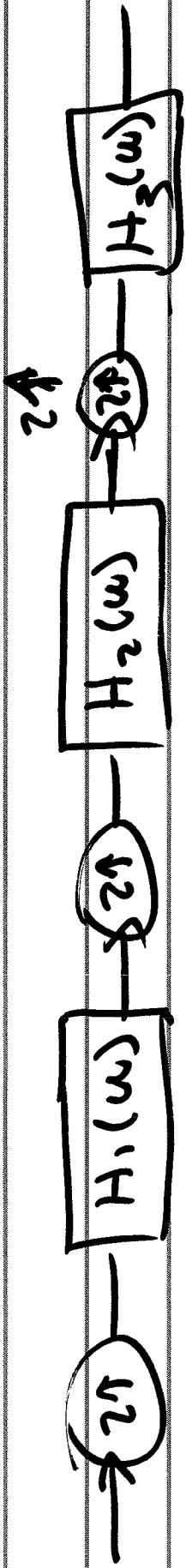


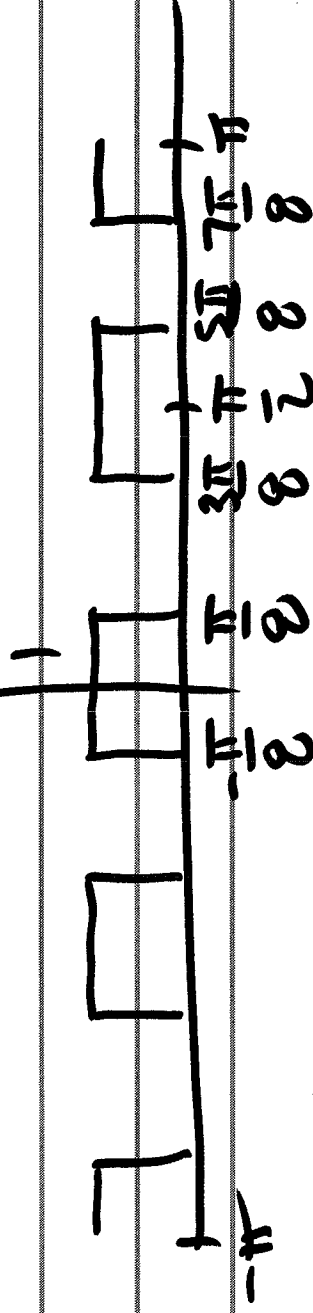
# Prob. 2 F'05

## FINAL EXAM



$$H_2(2w) H_1 H_3(w) = H_3(w)$$

$H_1(4\omega)$



$$H_1(4\omega)H_2(2\omega)H_3(\omega) = H_3(\omega)$$

$$S_{xx}(\omega) = |H_a(\omega)|^2 \underbrace{S_{yy}(\omega)}_{\delta(\omega)}$$

$$= H_3(\omega)$$

$$= H_{BP}(\omega)$$

$$r_{xx} [m] = 2 \cos\left(\frac{\pi}{2} m\right) \sin\left(\frac{\pi}{8} m\right)$$

$$a_1(1) = -\frac{r_{xx}(1)}{r_{xx}(0)} = 0 = \frac{1}{4} \sqrt{2(-1)} \frac{1}{\sqrt{2}}$$

$$E_{\min}^{(1)} = r_{xx}[0]$$

$$\begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = - \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$a_2(1) = 0 \quad a_2(2) = \frac{4}{\pi \sqrt{2}} \left. \begin{aligned} E_{\min}^{(2)} = E_{\min}^{(1)} (1 - a_2^2(2)) = \frac{1}{4} \left\{ 1 - \left( \frac{8}{\pi^2} \right) \right\} \end{aligned} \right\}$$

Prob. 3 F'05 FINAL EXAM

$$h[n] = -a_1 h[n-1] - a_2 h[n-2]$$

$$-b_0 \delta[n] - b_1 \delta[n-1]$$

$$h[2] = -a_1 h[1] - a_2 h[0]$$

$$h[3] = -a_1 h[2] - a_2 h[1]$$

$$\begin{bmatrix} 13 & 35 \\ 5 & 13 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$p_1 = 1/2$$

$$p_2 = 1/3$$

$$H(z) = \frac{-b_0 - b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Prob. 4 F' 05

FINAL EXAM

$$\begin{bmatrix} 2 & -j\sqrt{2} \\ j\sqrt{2} & 2 \end{bmatrix} \begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = - \begin{bmatrix} j\sqrt{2} \\ 0 \end{bmatrix}$$

$$z^2 + a_2(1)z + a_2(2)$$

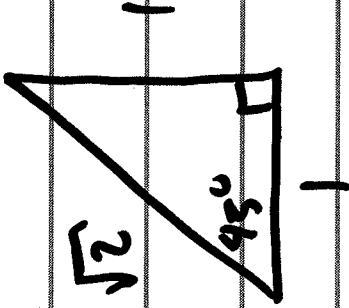
$$= (z - e^{j\omega_1})(z - e^{j\omega_2})$$

$$a_2(1) = \frac{\begin{vmatrix} -j\sqrt{2} & -j\sqrt{2} \\ 0 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -j\sqrt{2} \\ j\sqrt{2} & 2 \end{vmatrix}} = \frac{-j2\sqrt{2}}{4-2} = -j\sqrt{2}$$

$$j\sqrt{2}(-j\sqrt{z}) + 2a_2(z) = 0$$

$$a_2(z) = -1$$

$$z^2 - j\sqrt{2}z - 1 = 0$$

$$z = \frac{j\sqrt{2} \pm \sqrt{-2 + 9}}{2}$$


$$= \pm \frac{j\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} =$$

$$+ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \quad - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$e^{j\frac{\pi}{4}} \quad e^{j\frac{3\pi}{4}}$$

$$\omega_1 = \pi/4$$

$$\omega_2 = 3\pi/4$$

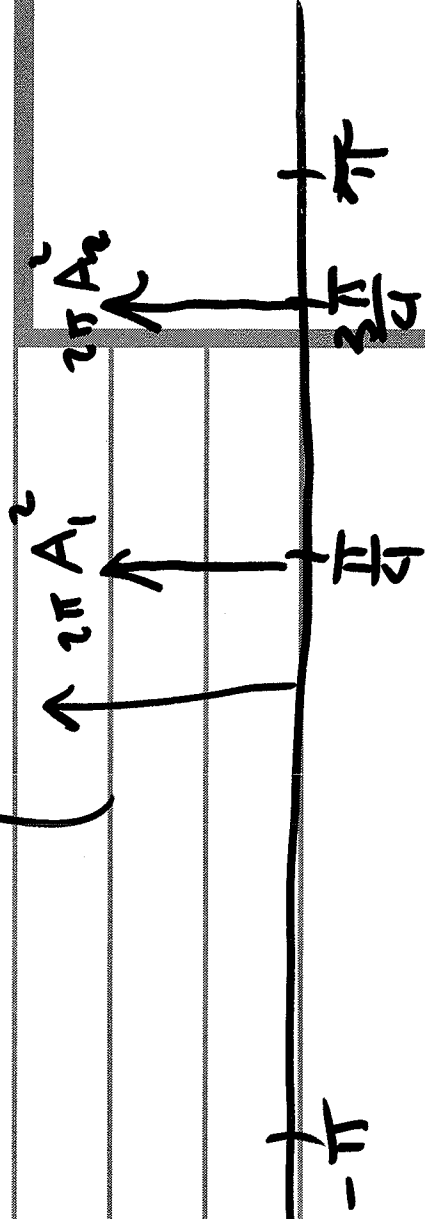
$$r_{xx}[3] = -a_2(1) r_{xx}[2] - a_2(2) r_{xx}[1]$$

$$= j\sqrt{2}(0) + 1(j\sqrt{2})$$

$$= j\sqrt{2}$$

$$S_{xx}(\omega) = \mathcal{F}\{r_{xx}[n]\} = ?$$

$$= \mathcal{F}\{A_1^2 e^{j\omega_1 n} + A_2^2 e^{j\omega_2 n}\}$$

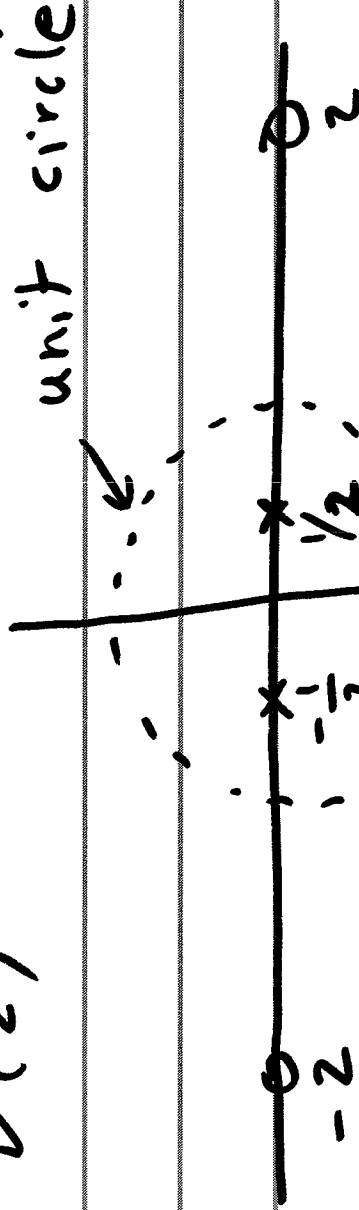


# Prob. 5 Final F'05

$$x[n] = \frac{1}{4} x[n-2] + v[n] - 4v[n-2]$$

$$\frac{X(z)}{V(z)} = \frac{1 - 4z^{-2}}{1 - \frac{1}{4}z^{-2}} = \frac{z^2 - 4}{z^2 - \frac{1}{4}} = \frac{(z+2)(z-2)}{(z+\frac{1}{2})(z-\frac{1}{2})}$$

unit circle



all-pass

filter

$$H(0) = \frac{1^2 - 4}{1^2 - \frac{1}{4}} = \frac{-3}{\frac{3}{4}} = -4$$

$$|H(\omega)|^2 = 16$$

$\forall \omega$



$$(b) S_{xx}(\omega) = |H(\omega)|^2 \int_{xx} S_{xx}(\omega)$$

$$= 16 \quad (2)$$

$$= 32 \quad (4)$$

$$(c) r_{xx} [m] = 32 \quad (m)$$

$$(d) a_1(1) = 0 = \frac{r_{xx} [1]}{r_{xx} [0]} = \frac{[1]}{[0]}$$

$$(e) \sigma_{min}^{(1)} = r_{xx} [0] = 32$$

$$(f) a_2(1) = 0, a_2(2) = 0$$

$$\sigma_{min}^{(2)} = 32$$