EE538 Final Exam Fall 2004
10:20 am -12:20 pm POTR Studio Dec. 13, 2004

Cover Sheet

Test Duration: 120 minutes.
Open Book but Closed Notes.
Calculators **ARE** allowed!!
This test contains **five** problems.
Each of the **five** problems are equally weighted.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Problem 1.

Consider a DT LTI system whose impulse response is

$$h[n] = \left(\frac{1+j}{\sqrt{2}}\right)^n u[n]$$

(a) Is the system BIBO stable? Substantiate your answer mathematically.

(b) Find a bounded input signal $x[n]$ that produces an unbounded output from this system.

(c) Find the system transfer function $H(z)$ of this system and draw the pole-zero diagram.

(d) Write the difference equation for the LTI system having the impulse response above.

(e) Plot a rough sketch of the magnitude of the DTFT of $h[n]$, $|H(\omega)|$, over $-\pi < \omega < \pi$, showing as much detail as possible.

(f) Consider the input signal below which is a sum of sinewaves "turned on" for all time.

$$x[n] = 1 + (j)^n + (-1)^n + \left(\frac{1-j}{\sqrt{2}}\right)^n$$

Write a closed-form expression for the corresponding output $y[n]$. ALSO, plot a rough sketch of the magnitude of the DTFT of $y[n]$, $|Y(\omega)|$, over $-\pi < \omega < \pi$, showing as much detail as possible.

(g) Let $y[n]$ denote the output obtained with the input signal below relative to the LTI system with impulse response above.

$$x[n] = (0.5)^n u[n]$$

Write a closed-form expression for the cross-correlation $r_{yx}[\ell]$ between the output $y[n]$ and the input $x[n]$.

Problem 2.

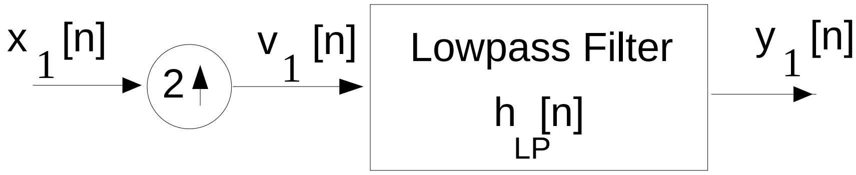

Figure 1.

Let $x_1[n]$ be a DT signal equal to a sum of sinwaves "turned on" for all time.

$$x_1[n] = 1 + (-j)^n + (j)^n$$

$x_1[n]$ is input to the system above, where the impulse response of the lowpass filter is

$$h_{LP}[n] = \left\{ \frac{\sin(\frac{2\pi}{3}n)}{\pi n} \right\} + \left\{ \frac{\sin(\frac{\pi}{3}n)}{\pi n} \right\}, \qquad -\infty < n < \infty,$$

The zero inserts may be mathematically described as $v_1[n] = \sum_{k=-\infty}^{\infty} x_1[k]\delta[n-2k]$.

(a) Plot the magnitude of the DTFT of the impulse response of the lowpass filter $h_{LP}[n]$, $H_{LP}(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible. This lowpass filter has a "don't care" region where the gain rolls off linearly from 1 to 0. Does this have any impact on the upsampling process?

(b) Plot the magnitude of the DTFT of the output $y_1[n]$, $Y_1(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible. The edge frequencies of any rectangularly shaped spectral components need to be clearly indicated.

(c) The up-sampling by a factor of 2 in Figure 1 above can be efficiently done via the top half of the block diagram in Figure 2 at the top of the next page.

(i) Provide an analytical expression for $h_{LP}^{(0)}[n] = h_{LP}[2n]$ for $-\infty < n < \infty$. Simplify. Plot the magnitude of the DTFT of $h_{LP}^{(0)}[n]$, $|H_{LP}^{(0)}(\omega)|$, over $-\pi < \omega < \pi$.

(ii) Is $y_1^{(0)}[n] = x_1[n]$? Explain your answer.

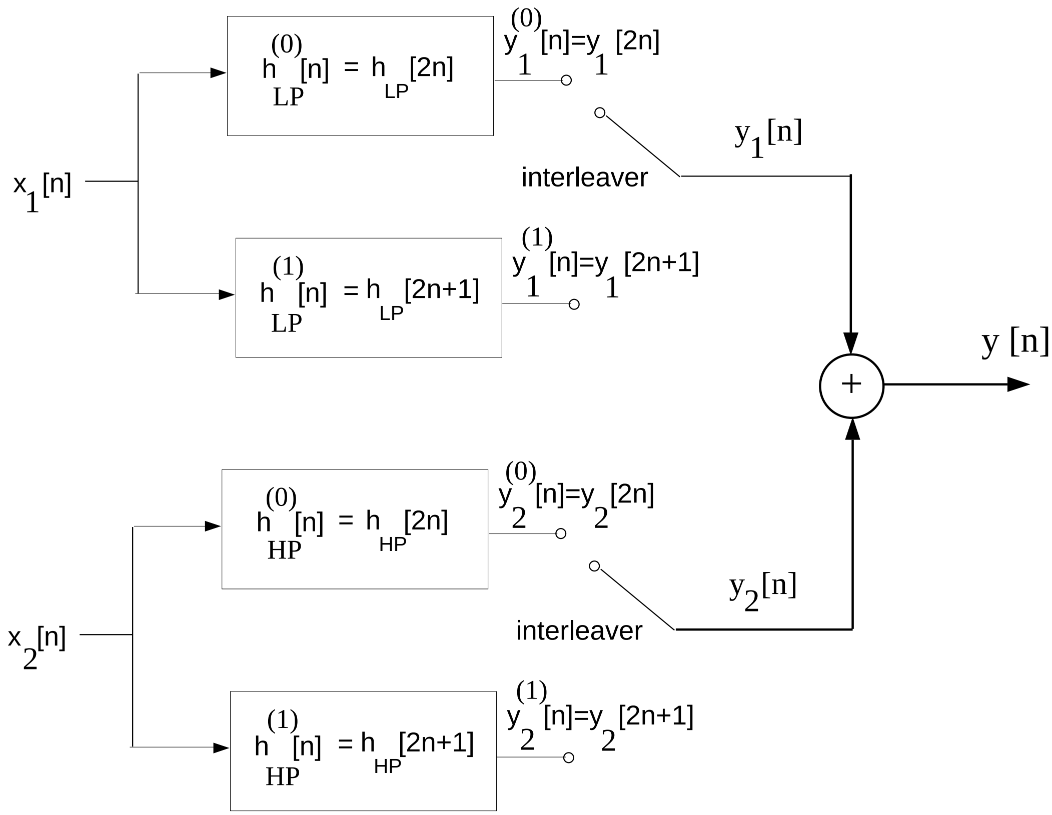

Figure 2.

Problem 3. Consider a second DT signal

$$x_2[n] = 2\left\{\frac{\sin(\frac{\pi}{6}n)}{\pi n}\right\}\cos\left(\frac{\pi}{2}n\right)$$

We desire to frequency division multiplex $x_1[n]$ (defined in Problem 2 on the previous page) and $x_2[n]$ directly above through digital subbanding as shown in Figure 2 above, where the highpass filter is

$$h_{HP}[n] = (-1)^n h_{LP}[n] = (-1)^n \left\{\left\{\frac{\sin(\frac{2\pi}{3}n)}{\pi n}\right\} + \left\{\frac{\sin(\frac{\pi}{3}n)}{\pi n}\right\}\right\} \qquad -\infty < n < \infty,$$

The sum signal is $y[n] = y_1[n] + y_2[n]$, where $y_1[n]$ is the same $y_1[n]$ created in Problem 2.

(a) Plot the magnitude of the DTFT of the sum signal $y[n] = y_1[n] + y_2[n]$. Show as much detail as possible. The respective frequencies of any sinusoidal components need to be clearly indicated as well as the edge frequencies of any rectangularly shaped spectral components.

(b) Draw a block diagram of a system to recover $x_1[n]$ from the sum signal $y[n] = y_1[n] + y_2[n]$. The recovery of $x_1[n]$ must be done in a computationally efficient manner. You CANNOT use any decimators and you CANNOT use any modulators (no multiplication by a sinewave). Clearly specify all the quantities in your block diagram.

(c) Draw a block diagram of a system to recover $x_2[n]$, from the sum signal $y[n] = y_1[n] + y_2[n]$. The same rules apply as those stated in part (b) above.

Problem 4.

A symmetric FIR lowpass filter of length $N = 5$ is characterized by the difference equation below. The passband edge is $\omega_p = \frac{\pi}{10}$ and the stopband edge is $\omega_s = \frac{3\pi}{10}$. NOTE that this filter is not equi-ripple and that calculators may be used for this problem.

$$y[n] = \frac{1}{5}x[n] + \frac{1}{5}x[n-1] + \frac{1}{5}x[n-2] + \frac{1}{5}x[n-3] + \frac{1}{5}x[n-4] \tag{1}$$

(a) Plot the magnitude of the frequency response of this filter over $-\pi < \omega < \pi$. Show as much detail as possible. (Recall that the frequency response is the DTFT of the impulse response of the filter.)

(b) Plot the phase of the frequency response of this filter over $-\pi < \omega < \pi$.

(i) Is the phase linear over the passband?

(ii) What is the delay of this filter (in discrete time units)?

(c) The frequency response at $\omega = 0$ is one. What is the maximum deviation δ_1 from one over the passband?

(d) What is the maximum absolute deviation δ_2 from zero over the stopband?

Consider that we create a new filter by convolving the impulse response of the filter described by Equation (1) above with itself. This yields a new symmetric FIR filter described by the following difference equation:

$$y[n] = \frac{1}{25}x[n] + \frac{2}{25}x[n-1] + \frac{3}{25}x[n-2] + \frac{4}{25}x[n-3] + \frac{5}{25}x[n-4] + \frac{4}{25}x[n-5] + \frac{3}{25}x[n-6] + \frac{2}{25}x[n-7] + \frac{1}{25}x[n-8]$$

(f) Plot the magnitude of the frequency response of this new filter over $-\pi < \omega < \pi$. Show as much detail as possible.

(g) Plot the phase of the frequency response of the new filter over $-\pi < \omega < \pi$.

(i) Is the phase linear over the passband?

(ii) What is the delay of this new filter (in discrete time units)?

(h) The frequency response at $\omega = 0$ is one. For the new filter, what is the maximum deviation δ_1 from one over the passband? Recall that the passband edge is $\omega_p = \frac{\pi}{10}$.

(i) For the new filter, what is the maximum absolute deviation δ_2 from zero over the stopband? Recall that the stopband edge is $\omega_s = \frac{3\pi}{10}$.

Problem 5.

(a) Let $X_9(k) = X(2\pi k/9)$, where $X(\omega)$ is the DTFT of the sequence

$$x[n] = (0.9)^n u[n] \overset{DTFT}{\longleftrightarrow} X(\omega) = \frac{1}{1 - 0.9e^{-j\omega}}$$

That is, $X_9(k)$ is the frequency domain sequence obtained by sampling $X(\omega)$ at $N = 9$ equi-spaced points in the interval $0 \le \omega < 2\pi$. Theory derived in class and in the textbook dictates that the 9-pt inverse DFT of $X_9(k)$ may be expressed as

$$x_9[n] = \sum_{\ell=-\infty}^{\infty} x[n - \ell 9]\{u[n] - u[n-9]\} \underset{9}{\overset{DFT}{\longleftrightarrow}} X_9(k) = \frac{1}{1 - 0.9e^{-j2\pi k/9}}; k = 0, 1, ..., 8$$

Determine a simple, closed-form expression for $x_9[n]$. A *closed-form* expression contains NO summations and it is NOT a listing of numbers. *Hint:*

$$\frac{1}{1 - (.9)^9} = 1.6324$$

(b) Consider normalizing $x_9[n]$ so that it's first value is one, $\tilde{x}_9[n] = x_9[n]/x_9[0]$, $n = 0, 1, ..., 8$. Compare $\tilde{x}_9[n]$ and $x[n]$ over $n = 0, 1, ..., 8$. Are they the same or different? Briefly explain your answer as to why or why not they are the same.

(c) Let $Y_9(k) = Y(2\pi k/9)$, where $Y(\omega)$ is the DTFT of the sequence

$$y[n] = (0.9)^{|n-4|} \overset{DTFT}{\longleftrightarrow} Y(\omega) = e^{-j4\omega}\left\{\frac{1 - (.9)^2}{1 - 2(0.9)\cos(\omega) + (.9)^2}\right\}$$

That is, $Y_9(k)$ is the frequency domain sequence obtained by sampling $Y(\omega)$ at $N = 9$ equi-spaced points in the interval $0 \le \omega < 2\pi$. Theory derived in class and in the textbook dictates that the 9-pt inverse DFT of $Y_9(k)$ may be expressed as

$$y_9[n] = \sum_{\ell=-\infty}^{\infty} y[n - \ell 9]\{u[n] - u[n-9]\} \underset{9}{\overset{DFT}{\longleftrightarrow}} Y_9(k) = \left\{\frac{1 - (.9)^2}{1 - 2(0.9)\cos(2\pi k/9) + (.9)^2}\right\} e^{-j8\pi k/9}$$

where $k = 0, 1, ..., 8$ for the right hand side directly above. Determine a simple, closed-form expression for $y_9[n]$. A *closed-form* expression contains NO summations and it is NOT a listing of numbers.

(d) Consider normalizing $y_9[n]$ so that it's first value is one, $\tilde{y}_9[n] = y_9[n]/y_9[0]$, $n = 0, 1, ..., 8$. Compare $\tilde{y}_9[n]$ and $y[n]$ over $n = 0, 1, ..., 8$. Are they the same or different? Briefly explain your answer as to why or why not they are the same.