EE538 **Final Exam** **Fall 2002**
Digital Signal Processing I **9 December 2002**

Cover Sheet

Test Duration: 2 hours.
Open Book but Closed Notes.
Calculators allowed (but not necessary).
This test contains **five** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

No.	**Topic(s) of Problem**	**Points**
1.	Multi-Stage Up-Sampling	20
2.	Principles of Upsampling and Downsampling	20
3.	DFT and Properties	20
4.	AR/ARMA Spectral Estimation	20
5.	Sum of Sinewaves Spectral Analysis	20

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GIVEN NOBLE's IDENTITIES TO USE IN PROBLEM 1.

(a) If $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

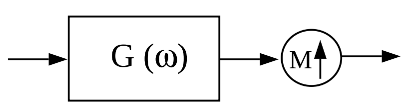


Figure 1(a).

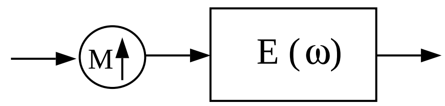


Figure 1(b).

(b) If $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) satisfies $F(\omega) = H(M\omega)$, the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

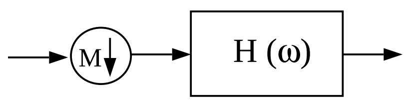


Figure 2(a).

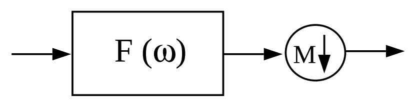


Figure 2(b).

Problem 1. [20 points]

(a) Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$. *Hint:* Analyze the system of Figure 3(a) in the frequency domain using Noble's First Identity.

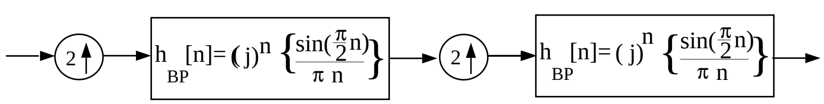


Figure 3(a).

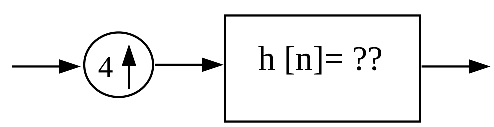


Figure 3(b).

(b) Determine the numerical values of the impulse response $h_{eq}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). *Hint:* Analyze the system of Figure 4(a) in the time domain using Noble's First Identity.

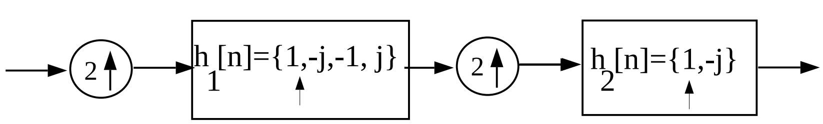


Figure 4(a).

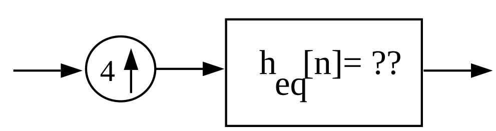
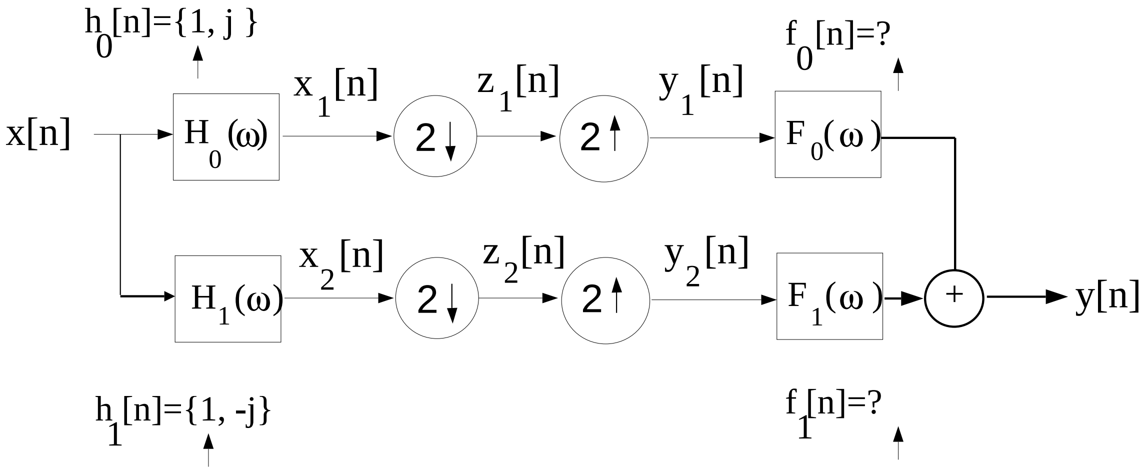


Figure 4(b).

Problem 2. [20 points]

In the system below, each of the analysis filters, $h_0[n]$ and $h_1[n]$, and each of the two synthesis filters, $f_0[n]$ and $f_1[n]$, is a causal FIR filter of length 2. The specific values of $h_0[n]$ and $h_1[n]$ are indicated. The arrow denotes the value at $n = 0$. (See the hints at the bottom of the page.)



where $z_i[n] = x_i[2n], \quad i = 1, 2$ and $y_i[n] = \sum_{\ell=-\infty}^{\infty} z_i[\ell]\delta[n - 2\ell], \quad i = 1, 2$. Determine the numerical values of $f_0[n], n = 0, 1$ and $f_1[n], n = 0, 1$, such that $y[n] = 2x[n - 1]$ for any input sequence $x[n]$.

Hint 1. The series combination of a downsampler followed by an upsampler does NOT reduce to an identity transformation – they don't "cancel" each other.

Hint 2. This problem can be solved either in the time domain or the frequency domain with about equal complexity.

Problem 3. [20 points]

Consider an input sequence, $x[n]$, of length $L = 6$ and an FIR filter with impulse response $h[n]$ of length $M = 6$ as described below.

$$
\begin{aligned}
x[n] &= u[n] - u[n-6] = \{1, 1, 1, 1, 1, 1\} \\
h[n] &= u[n] - u[n-6] = \{1, 1, 1, 1, 1, 1\}
\end{aligned}
$$

We compute an $N = 8$-pt. DFT of each of these two sequences as

$$
x[n] \overset{\text{DFT}}{\underset{8}{\longleftrightarrow}} X_8[k] \qquad\qquad h[n] \overset{\text{DFT}}{\underset{8}{\longleftrightarrow}} H_8[k]
$$

Next, we point-wise multiply the DFT sequences to form $Y_8[k] = X_8[k]H_8[k]$, $k = 0, 1, ..., 7..$ Finally, we compute an $N = 8$-pt. inverse DFT of $Y_8[k]$ to obtain $y_P[n]$. Determine the numerical values of $y_P[n]$ for $n = 0, 1, 2, 3, 4, 5, 6, 7..$ **You can solve the problem any way you like but briefly explain how you got your answer. Actually computing the DFT's is NOT the way to solve this problem.**

Problem 4. [20 points]

Consider the ARMA(1,1) process generated via the difference equation

$$x[n] = -\frac{1}{2}x[n-1] + w[n] - w[n-1]$$

where $w[n]$ is a stationary white noise process with variance $\sigma_w^2 = 1$.

(a) Determine the numerical values of $r_{xx}[0]$, $r_{xx}[1]$, $r_{xx}[2]$, where $r_{xx}[m]$ is the autocorrelation sequence $r_{xx}[m] = E\{x[n]x[n-m]\}$. (Note that $r_{xx}[m]$ is the inverse DTFT of the spectral density $S_{xx}(\omega)$ asked for in Part (b) below, but there at least three different ways you can solve this part of the problem.)

(b) Determine a simple closed-form expression for the spectral density for $x[n]$, $S_{xx}(\omega)$, which may be expressed as the DTFT of $r_{xx}[m]$:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}[m]e^{-jm\omega}$$

(c) Consider the first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical value of the optimum predictor coefficient $a_1(1)$ and the corresponding minimum mean-square error.

Problem 5. [20 points]

Consider the discrete-time complex-valued random process defined for all n:

$$x[n] = D + A_1 e^{\jmath(\omega_1 n + \Theta_1)} + A_2 e^{\jmath(\omega_2 n + \Theta_2)} + \nu[n]$$

where the respective frequencies, ω_1 and ω_2, of the two complex sinewaves are deterministic but unknown constants. The amplitudes, A_1 and A_2, and the constant D are also deterministic but unknown constants. Θ_1 and Θ_2 are independent random variables with each uniformly distributed over a 2π interval and $\nu[n]$ is a stationary random process with zero mean and $r_{\nu\nu}[m] = E\{\nu[n]\nu^*[n-m]\} = \delta[m]$. That is, $\nu[n]$ forms an i.i.d. sequence with a variance of unity. Note, $\nu[n]$ is independent of both Θ_1 and Θ_2 for all n. The values of the autocorrelation sequence for $x[n]$, $r_{xx}[m] = E\{x[n]x^*[n-m]\}$, for three different lag values are given below.

$$r_{xx}[0] = 5, \quad r_{xx}[1] = -1 + j, \quad r_{xx}[2] = 2, \quad r_{xx}[3] = -1 - j$$

(a) Determine the numerical values of ω_1 and ω_2. **You have to use what you've learned during the parametric spectral analysis portion of this course. You will be given no credit if you simply set up a system of equations to solve based on the form of** $r_{xx}[m] = \sum_{i=1}^{p} A_i^2 e^{j\omega_i m}$ **and solve this nonlinear system of equations.**

(b) Consider a first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical values of the optimum predictor coefficient $a_1(1)$, and the numerical value of the corresponding minimum mean-square error.

(c) Consider a second-order predictor

$$\hat{x}[n] = -a_2(1)x[n-1] - a_2(2)x[n-2]$$

Determine the numerical values of the optimum predictor coefficients $a_2(1)$ and $a_2(2)$, and the numerical value of the corresponding minimum mean-square error.

(d) Consider a third-order predictor

$$\hat{x}[n] = -a_3(1)x[n-1] - a_3(2)x[n-2] - a_3(3)x[n-3]$$

Determine the numerical values of the optimum predictor coefficients $a_3(1)$, $a_3(2)$ and $a_3(3)$, and the numerical value of the corresponding minimum mean-square error.