Exam 1 F2001 EE538

Sol'n to Prob. 1

(a) $y_1[n] = x[n] * h_1[n]$

$y_2[n] = x[n] * h_2[n]$

• Thus, $y_1[n] * h_2[n] = x[n] * h_1[n] * h_2[n]$

(convolution is commutative) $= x[n] * h_2[n] * h_1[n]$

(convolution is associative) $= y_2[n] * h_1[n]$

$\Rightarrow y_1[n] * h_2[n] - y_2[n] * h_1[n] = 0$ (1)

If $x[n]$, $h_1[n]$, and $h_2[n]$ are all causal, $y_1[n]$ and $y_2[n]$ are causal such that (1) may be expressed as

$$\sum_{k=0}^{1} h_2[k]\, y_1[n-k] - \sum_{k=0}^{1} h_1[k]\, y_2[n-k] = 0$$

$$h_2[0]\, y_1[n] + h_2[1]\, y_1[n-1] - h_1[0]\, y_2[n] - h_1[1]\, y_2[n-1] = 0$$

- Rearranging, and since given $h_1[0] = 1$:

$$-h_1[1]\, y_2[n-1] + h_2[0]\, y_1[n] + h_2[1]\, y_1[n-1] = y_2[n]$$

- need 3 eqns in 3 unknowns:

$$\begin{array}{l} n=0 \\ n=1 \\ n=2 \end{array} \begin{bmatrix} 0 & 1 & 0 \\ -3 & 4 & 1 \\ -5 & 7 & 4 \end{bmatrix} \begin{bmatrix} h_1[1] \\ h_2[0] \\ h_2[1] \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

$h_2[0] = 3$; $h_1[1] = 2$; $h_2[1] = -1$

$h_1[n] = \{1, 2\}$ $h_2[n] = \{3, -1\}$

Sol'n. to Prob. 2

(a) Examine $H(z)$ for FIR filter:

$$\frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} e^{j2\pi \frac{\ell}{N} n} z^{-n} = H(z)$$

$$= z^{-(N-1)} \left\{ \sum_{n=0}^{N-1} e^{j2\pi \frac{\ell}{N} n} z^{n} \right\}$$

$$= z^{-(N-1)} \sum_{n=0}^{N-1} \left(z e^{j2\pi \frac{\ell}{N}} \right)^{n}$$

Recall $\displaystyle\sum_{n=0}^{N-1} e^{j2\pi \frac{k}{N} n} = 0$ except when $k=0$

Thus, zeroes are such that

$$z_k e^{j2\pi \frac{\ell}{N}} = e^{j2\pi \frac{k}{N}} \qquad k = 1, 2, \ldots, N-1$$

$$z_k = e^{-j2\pi \frac{\ell}{N}} e^{j2\pi \frac{k}{N}}$$

$$= e^{j2\pi \frac{(k-\ell)}{N}} \qquad k = 1, 2, \ldots, N-1$$

zeroes equi-spaced around unit circle at angles $m\frac{2\pi}{N}$ except $m = +\ell$ is missing

$\Rightarrow$ there is not a zero at $e^{+j2\pi \frac{\ell}{N}}$

The idea is to put that zero in and divide by it as well since the roots of $z^N - 1 = 0$ are $z_m = e^{j2\pi \frac{m}{N}}$, $m = 0, 1, \ldots, N-1$

Sol'n. to Prob. 2 (cont.)

Examine IIR filter for $D = N$

IIR: $$\frac{Y(z)}{X(z)} = \frac{1 - z^{-D}}{1 - a_1 z^{-1}} = \frac{1 - z^{-N}}{1 - a_1 z^{-1}} \left(\frac{z^N}{z^N} \right)$$

$$= z^{-(N-1)} \frac{z^N - 1}{z - a_1}$$

$\Rightarrow$ zeroes at $e^{j 2\pi \frac{m}{N}}$, $m = 0, 1, \ldots, N-1$

" pole at $z = a_1$

Compare to FIR:

$$H(z) = z^{-(N-1)} \prod_{\substack{m=0 \\ m \neq \ell}}^{N-1} \left(z - e^{j 2\pi \frac{m}{N}} \right) \qquad \underbrace{\frac{\left(z - e^{j 2\pi \frac{\ell}{N}}\right)}{\left(z - e^{j 2\pi \frac{\ell}{N}}\right)}}_{\text{pole - zero cancellation}}$$

$$= z^{-(N-1)} \frac{\prod_{m=0}^{N-1} \left(z - e^{j 2\pi \frac{m}{N}} \right)}{z - e^{j 2\pi \frac{\ell}{N}}}$$

$$= z^{-(N-1)} \frac{\left(z^N - 1 \right)}{z - e^{j 2\pi \ell / N}}$$

Answers to (a):

$a_1 = e^{j 2\pi \frac{\ell}{N}}$ $\quad D = N$

Sol'n. to Prob. 2 (cont.)

(b) $\ell = 2$ and $N = 4$

(i) $a_1 = e^{j 2\pi \frac{2}{4}} = -1$

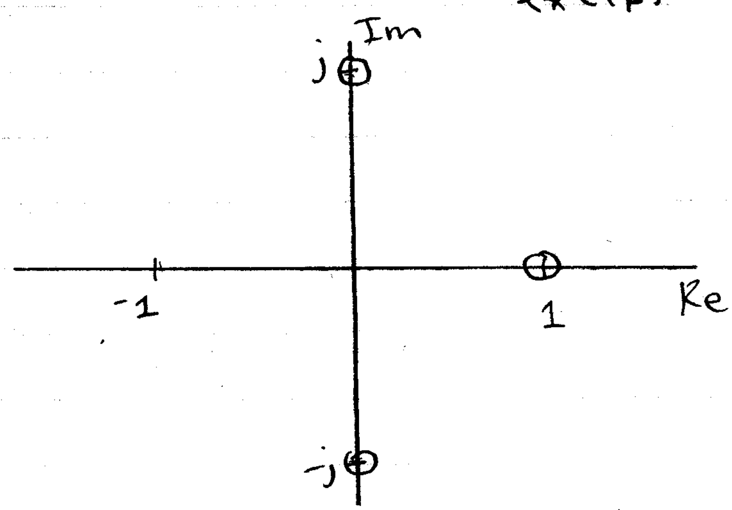
$D = 4$

$$h[n] = \left\{ 1,\ e^{-j 2\pi \frac{(2)}{4}},\ e^{-j 2\pi \frac{(2)(2)}{4}},\ e^{-j 2\pi \frac{(2)(3)}{4}} \right\}$$

$$= \{\underset{\uparrow}{1},\ -1,\ 1,\ -1\}$$

(ii) Zeroes at $z_m = e^{j 2\pi \frac{m}{4}}$

except at $m = \ell = 2$



ROC is everywhere except at $z = 0$

(iii) This is a <u>highpass</u> filter

$\Rightarrow$ no zero at $z = -1$ corresponding to $\omega = \pi$

(iv) $h[n] = \{\underset{\uparrow}{1}, -1, 1, -1\}$