Example Test Question for Exam 1 on Autocorrelation Material

$x_1[n]$ and $x_2[n]$ are said to form a complementary pair if

\[ r_{x_1x_1}[l] + r_{x_2x_2}[l] = \delta[l] \]

where:

\[ r_{x_ix_i}[l] = x_i[l] \star x_i^*[l-\ell] \]

\(i = 1, 2\)

Question: Do 

\[ y_1[n] = e^{j\omega_0 n} x_1[n] \]

and 

\[ y_2[n] = e^{j\omega_0 n} x_2[n] \] form a complementary pair? For any \(\omega_0\), justify your answer with analysis.
In notes call Additional Properties of Autocorrelation, we proved that if

\[ y[n] = e^{j(\omega_0 n + \theta)} x[n] \] then

\[ r_{yy}[k] = e^{j\omega_0 k} r_{xx}[k] \] Thus, for this problem \[ r_{y_1y_1}[k] = e^{j\omega_0 l} r_{x_1x_1}[k] \] \[ i = 1, 2 \]

Thus:

\[ r_{y_1y_1}[k] + r_{y_2y_2}[k] = e^{j\omega_0 l} r_{x_1x_1}[k] + e^{j\omega_0 l} r_{x_2x_2}[k] \]

\[ = e^{j\omega_0 l} \{ r_{x_1x_1}[k] + r_{x_2x_2}[k] \} = e^{j\omega_0 l} \delta[k] \]

\[ = e^{j\omega_0 l} \delta[k] = \delta[k] \] YES