Example Test Question for Exam 1 on Autocorrelation Material

$x_1[n]$ and $x_2[n]$ are said to form a complementary pair if

prop to:

$$r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell] = \delta[\ell]$$

where: $r_{x_ix_i}[\ell] = x_i[\ell] * x_i^*[-\ell]$ $\quad i = 1, 2$

Question: Do $y_1[n] = e^{j\omega_0 n} x_1[n]$ and $y_2[n] = e^{j\omega_0 n} x_2[n]$ form a complementary pair? for any ω_0

Justify your answer with analysis

• In notes call Additional Properties of Autocorrelation, we proved that if $y[n] = e^{j(\omega_0 n + \theta)} x[n]$, then $r_{yy}[\ell] = e^{j\omega_0 \ell} r_{xx}[\ell]$ Thus, for this problem $r_{y_i y_i}[\ell] = e^{j\omega_0 \ell} r_{x_i x_i}[\ell]$ $i = 1, 2$

Thus:

$$r_{y_1 y_1}[\ell] + r_{y_2 y_2}[\ell] = e^{j\omega_0 \ell} r_{x_1 x_1}[\ell] + e^{j\omega_0 \ell} r_{x_2 x_2}[\ell]$$

$$= e^{j\omega_0 \ell} \{ r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell] \} = e^{j\omega_0 \ell} \delta[\ell]$$

$$= e^{j\omega_0 (0)} \delta[\ell] = \delta[\ell]$$ YES

$$z[n] = a\,x[n] + b\,y[n]$$

$r_{zz}[\ell] = ?$ Use FOIL!

$$r_{zz}[\ell] = \{a\,x[\ell] + b\,y[\ell]\} * \{a^* x^*[-\ell] + b^* y^*[-\ell]\}$$

$$= |a|^2 r_{xx}[\ell] + |b|^2 r_{yy}[\ell]$$

$$+ a\,b^*\, r_{xy}[\ell] + b\,a^*\, r_{yx}[\ell]$$

where: $r_{xy}[\ell] = x[\ell] * y^*[-\ell]$

$r_{yx}[\ell] = y[\ell] * x^*[-\ell]$