

Example Test Question for Exam 1 on Autocorrelation Material

$x_1[n]$ and $x_2[n]$ are said to form a complementary pair if

$$r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell] = \delta[\ell]$$

where: $r_{x_i x_i}[\ell] = x_i[\ell] * x_i^*[-\ell]$
 $i=1,2$

Question: Do $y_1[n] = e^{j\omega_0 n} x_1[n]$

and $y_2[n] = e^{j\omega_0 n} x_2[n]$ form a

complementary pair? for any ω_0
Justify your answer with analysis

• In notes call Additional Properties of Autocorrelation, we proved that if

$$y[n] = e^{j(\omega_0 n + \theta)} x[n], \text{ then}$$

$$r_{yy}[\ell] = e^{j\omega_0 \ell} r_{xx}[\ell] \quad \text{Thus, for this}$$

$$\text{Problem } r_{y_i y_i}[\ell] = e^{j\omega_0 \ell} r_{x_i x_i}[\ell] \quad i=1,2$$

Thus:

$$r_{y_1 y_1}[\ell] + r_{y_2 y_2}[\ell] = e^{j\omega_0 \ell} r_{x_1 x_1}[\ell] + e^{j\omega_0 \ell} r_{x_2 x_2}[\ell]$$

$$= e^{j\omega_0 \ell} \{r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell]\} = e^{j\omega_0 \ell} \delta[\ell]$$

$$= e^{j\omega_0(0)} \delta[\ell] = \delta[\ell] \quad \text{YES}$$