

SOLUTION

NAME:
Digital Signal Processing I **Exam 3** **2018**
Session 40 **Fall 2018**
30 Nov. 2018

Cover Sheet

Test Duration: 60 minutes.

Open Book but Closed Notes. One 8.5 x 11 crib sheet allowed
Calculators NOT allowed.

Show all work. More credit for approach than final answer.

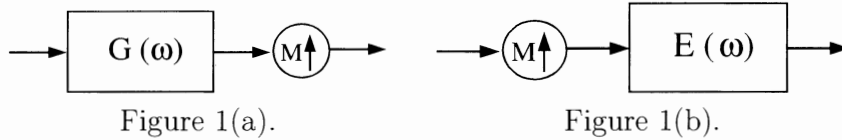
This test contains **THREE** problems.

All work should be done on the blank pages provided.

Your answer to each part of the exam should be clearly labeled.

GIVEN NOBLE'S IDENTITIES TO USE IN PROBLEM 1.

- (a) If $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) satisfies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

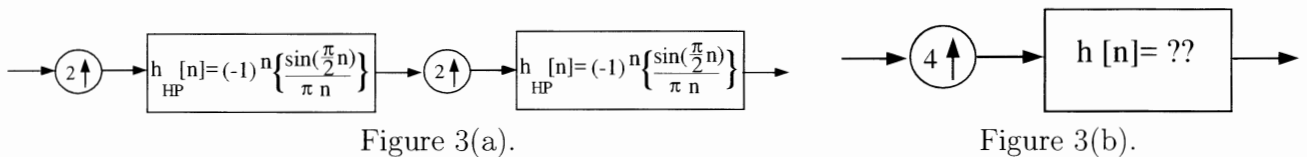


- (b) If $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) satisfies $F(\omega) = H(M\omega)$, the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

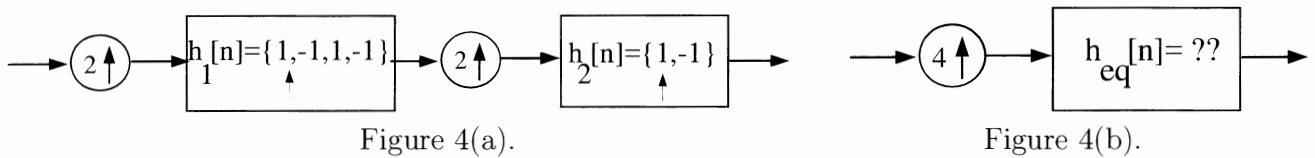


Problem 1.

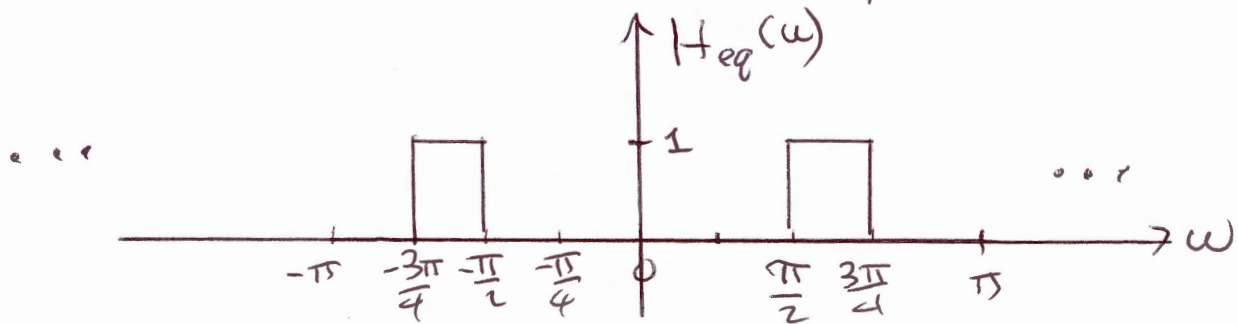
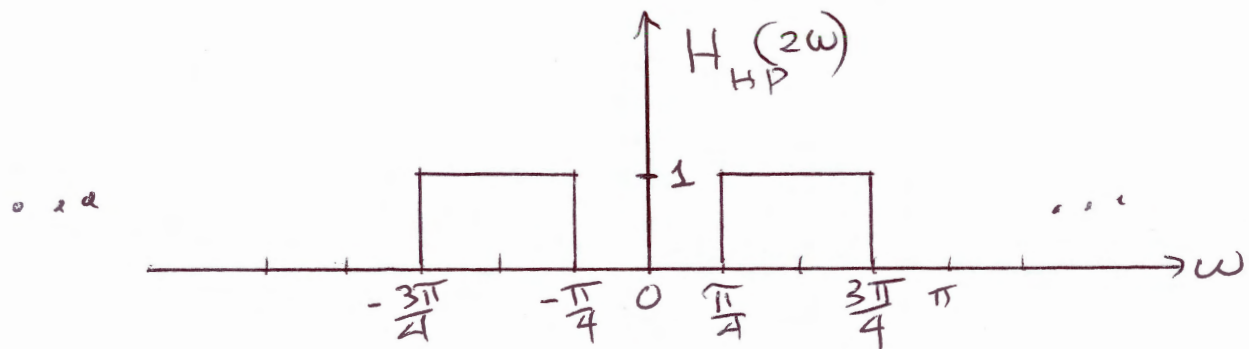
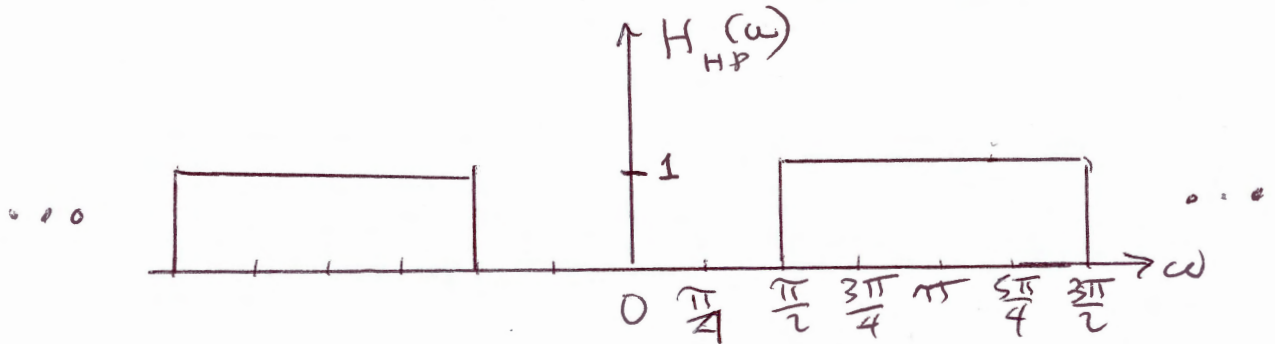
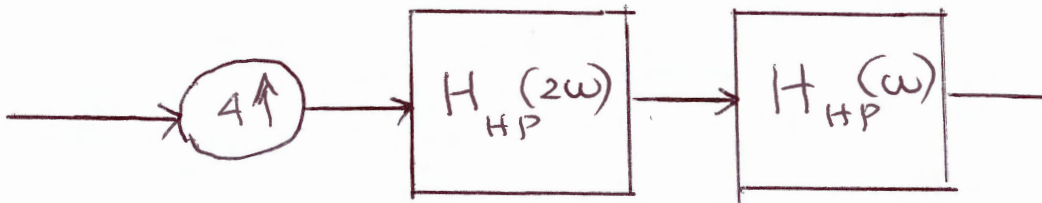
- (a) Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$. *Hint:* Analyze the system of Figure 3(a) in the frequency domain using Noble's First Identity.



- (b) Determine the numerical values of the impulse response $h_{eq}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). *Hint:* Analyze the system of Figure 4(a) in the time domain using Noble's First Identity.



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$$h_{eq}[n] = h[n] = 2 \frac{\sin\left(\frac{11}{8}n\right)}{\pi n} \cos\left(\frac{5\pi}{8}n\right)$$

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1 (b) Same procedure as in part (a)

$$H_{eq}(w) = H_1(2w) H_2(w)$$

$$h_{eq}[n] = \left\{ \sum_{k=0}^3 h_1[k] \delta[n-2k] \right\} * h_2[n]$$

$$h_{eq}[n] = \{1, 0, -1, 0, 1, 0, -1\} * \{1, -1\}$$

$$= \begin{array}{cccccccc} 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 \end{array}$$

$$h_{eq}[n] = \{1, -1, -1, 1, 1, -1, -1, 1\}$$

↑

Problem 2. Consider a causal FIR filter of length $M = 8$ with impulse response as defined below:

$$h[n] = 2 \sin\left(\frac{3\pi}{4}n\right) \{u[n] - u[n-8]\}$$

Consider a DT sinewave $x[n]$ of length $N = 16$ as defined below:

$$x[n] = \left\{ \cos\left(\frac{\pi}{2}n\right) + \cos\left(\frac{12\pi}{16}n\right) \right\} \{u[n] - u[n-16]\}$$

$y[n]$ is formed as the linear convolution of $x[n]$ with $h[n]$ as:

$$y[n] = x[n] * h[n]$$

We then take the last $M - 1 = 7$ values of $y[n]$ and time-domain alias them into the first three values to form a sequence of length 16, denoted $y_a[n]$, according to:

$$y_a[n] = y[n] + y[n+16], n = 0, 1, 2, \dots, 6 \quad (1)$$

$$y_a[n] = y[n], n = 7, 8, 9, 10, 11, 12, 13, 14, 15 \quad (2)$$

Determine an expression for $y_a[n]$ similar to the expression for $x[n]$ above. Show all work. You do NOT have to list the 16 numerical values of $y_a[n], n = 0, 1, \dots, 15$ in sequence form.

NOTE 1: Using concepts learned in class; There won't be a lot of points for brute force calculations that do not show what you learned in class for Exam 3.

$$y_a[n] = \left| H\left(\frac{\pi}{2}\right) \right| \cos\left(\frac{\pi}{2}n + \angle H\left(\frac{\pi}{2}\right)\right) \left\{ u[n] - u[n-16] \right\} \\ + \left| H\left(\frac{3\pi}{4}\right) \right| \cos\left(\frac{3\pi}{4}n + \angle H\left(\frac{3\pi}{4}\right)\right) \left\{ u[n] - u[n-16] \right\}$$

$$\left. \begin{aligned} \cos\left(\frac{\pi}{2}n\right) &= \cos\left(\left(4\right)\frac{2\pi}{16}n\right) & k_0 &= 4 \\ \cos\left(\frac{12\pi}{16}n\right) &= \cos\left(\left(6\right)\frac{2\pi}{16}n\right) & k_0 &= 6 \end{aligned} \right\} \begin{array}{l} \text{"lucky"} \\ \text{cases} \end{array}$$

$$= \cos\left(\frac{3\pi}{4}n\right)$$

$$h[n] \xleftrightarrow{\text{DTFT}} H(\omega) = \frac{1}{j} \frac{\sin\left(\frac{8}{2}\left(\omega - \frac{3\pi}{4}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega - \frac{3\pi}{4}\right)\right)} e^{-j \frac{(8-1)}{2}\left(\omega - \frac{3\pi}{4}\right)} \\ = \frac{1}{j} \frac{\sin\left(\frac{8}{2}\left(\omega + \frac{3\pi}{4}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega + \frac{3\pi}{4}\right)\right)} e^{-j \frac{(8-1)}{2}\left(\omega + \frac{3\pi}{4}\right)}$$

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$$H\left(\frac{3}{4}\pi\right) = -8j$$

$$H\left(\frac{\pi}{2}\right) = 0$$

$$y_a[n] = 8 \cos\left(\underbrace{\frac{3\pi}{4}n}_{\frac{12\pi}{16}} - \frac{\pi}{2}\right) \{u[n] - u[n-16]\}$$

Problem 3. Consider a causal FIR filter of length $M = 9$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \left\{ 2 \frac{\sin \left[\frac{\pi}{4} (n + \ell 9) \right]}{\pi (n + \ell 9)} - 4 \frac{\sin \left[\frac{\pi}{2} (n + \ell 9) \right]}{\pi (n + \ell 9)} + 3 \frac{\sin \left[\frac{3\pi}{4} (n + \ell 9) \right]}{\pi (n + \ell 9)} \right\} \{u[n] - u[n - 9]\}$$

(a) Determine the 9-pt DFT of $h_p[n]$, denoted $H_9(k)$, for $0 \leq k \leq 9$. Write your answer in sequence form to indicate the numerical values of $H_9(k)$, $k = 0, 1, \dots, 8$.

(b) Consider the sequence $x[n]$ of length $L = 9$ below.

$$x[n] = \left\{ -\cos \left(\frac{4\pi}{9} n \right) + 2 \sin \left(\frac{8\pi}{9} n \right) + \frac{1}{3} \cos \left(\frac{4\pi}{3} n \right) \right\} \{u[n] - u[n - 9]\}$$

$$\frac{4\pi}{3} = \frac{12\pi}{9}$$

$y_9[n]$ is formed by computing $X_9(k)$ as a 9-pt DFT of $x[n]$, $H_9(k)$ as a 9-pt DFT of $h_p[n]$ and, finally, then $y_9[n]$ is computed as the 9-pt inverse DFT of $Y_9(k) = X_9(k)H_9(k)$. Express the result $y_9[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

$$h_p[n] \xleftrightarrow[9]{\text{DFT}} \{1, 1, -1, 3, 0, 0, 3, -1, 0\}$$

k	0	1	2	3	4	5	6	7	8
ω	0	$\frac{2\pi}{9}$	$\frac{4\pi}{9}$	$\frac{6\pi}{9}$	$\frac{8\pi}{9}$	$\frac{10\pi}{9}$	$\frac{12\pi}{9}$	$\frac{14\pi}{9}$	$\frac{16\pi}{9}$
$H_p(k)$	1	1	-1	3	0	0	3	-1	0

$$y_9[n] = \underbrace{+}_{(-1)(-1)} \cos \left(\frac{4\pi}{9} n \right) + 0 \cdot 2 \sin \left(\frac{8\pi}{9} n \right) + 3 \left(\frac{1}{3} \right) \cos \left(\frac{12\pi}{9} n \right)$$

$$= \left\{ \cos \left(\frac{4\pi}{9} n \right) + \cos \left(\frac{4\pi}{3} n \right) \right\} \{u[n] - u[n - 9]\}$$

