ECE 538 Exam 3 Solution Fall '08

Prob. 1 $x[n] * h[n] = y[n]$

$L = 7$ $M = 7$ $M + L - 1 = 13$

$n = 0, 1, \ldots, 12$

look at 10 pt. DFT processing result: $y_{10}[n]$

$y_{10}[n] = y[n] + y[n+10]$

$13 - 10 = 3$ ⇒ 3 pts. at end aliased into 3 pts. at beginning

⇒ missing 6 values: $n = 0, 1, 2$
$10, 11, 12$

for $n = 3, \ldots, 9$: $y_{10}[n] = y[n]$

$y_{10}[0] = y[0] + y[10] = -2$

$y_{10}[1] = y[1] + y[11] = 0$

$y_{10}[2] = y[2] + y[12] = -2$

Similarly: $y_8[n] = y[n] + y[n+8]$, $n = 0, 1, \ldots, 7$

$\left.\begin{array}{l} y_8[0] = y[0] + y[8] = -2 \\ \text{from } y_{10}[n]: \ y[8] = -1 \end{array}\right\} \Rightarrow y[0] = -1$

$\left.\begin{array}{l} y_8[1] = y[1] + y[9] = 0 \\ \text{from } y_{10}[n]: \ y[9] = 0 \end{array}\right\} \Rightarrow y[1] = 0$

$y_8[2] = y[2] + y[10] = -2$ ⇒ $y[2] + (-1) = -2$

$y_{10}[0] = y[0] + y[10] = -2 = -1 + y[10] = -2$ $\left\}\begin{array}{l} y[2] = -1 \\ y[10] = -1 \end{array}\right.$

So far:

n:	0	1	2	3	4	5	6	7	8	9	10	11	12
$y[n] =$	-1	0	-1	0	-1	0	7	0	-1	0	-1	x	x

$$y[n] = \{-1, 0, -1, 0, -1, 0, 7, 0, -1, 0, -1, x, x\}$$

Only need to compute $y[11]$ and $y[12]$

$$\left.\begin{aligned} y_{10}[1] &= y[1] + y[11] = 0 \\ &\quad 0 + y[11] = 0 \end{aligned}\right\} y[11] = 0$$

$$\left.\begin{aligned} y_{10}[2] &= y[2] + y[12] = -2 \\ &\quad -1 + y[12] = -2 \end{aligned}\right\} y[12] = -1$$

(Autocorrelation of 7-length Barker code :))

Prob. 2 Solution

$x_1[n] = u[n] - u[n-6]$ length $L = 6$

$h[n]$ length $M=3$ $\Rightarrow y[n] = x_1[n] * h[n]$ is of length 8

Thus, 8 pt. DFT based processing produces linear convolution.

$$y_1[n] = x_1[n] * h[n] =$$

$$
\begin{array}{rl}
+ & \{-1,-1,-1,-1,-1,-1\} \\
 & \quad \{2,2,2,2,2,2\} \\
 & \quad\quad \{-1,-1,-1,-1,-1,-1\} \\
\hline
\end{array}
$$

(a) $y_1[n] = \{\underset{\uparrow}{-1}, 1, 0, 0, 0, 0, 1, -1\}$

(b) Since $x_2[n] = -x_1[n]$

$$y_2[n] = -y_1[n] = \{1, -1, 0, 0, 0, 0, -1, +1\}$$

(c) overlap is by $M-1 = 2$

$$
\begin{array}{l}
y[n] = \{-1, 1, 0, 0, 0, 0, 1, -1\} \\
\qquad\qquad\qquad\qquad\qquad \{1, -1, 0, 0, 0, 0, -1, 1\}
\end{array}
$$

$$= \{-1, 1, 0, 0, 0, 0, 2, -2, 0, 0, 0, 0, -1, 1\}$$

This filter is an example of a "unsharp mask" that enhances edges (makes them more pronounced) while ~~showing~~ producing zero output for "flat areas."

(d) $x_1[n]$, $x_2[n]$, and $h[n]$ are all real-valued

$$?? \quad \overset{DFT}{\underset{N}{\longleftrightarrow}} \quad \{X_1(k) + jX_2(k)\}\, H(k)$$

$$z[n] = \{x_1[n] + jx_2[n]\} \textcircled{N} h(n) \quad \overset{DFT}{\underset{N}{\longleftrightarrow}} \quad Z(k) = H(k)X_1(k) + H(k)X_2(k)$$

$$z(n) = x_1(n) \textcircled{N} h[n] + j x_2(n) \textcircled{N} h(n) \quad \overset{DFT}{\underset{N}{\longleftrightarrow}}$$

$$\mathrm{Re}\{Z(k)\} \neq H(k)X_1(k)$$

$$\mathrm{Im}\{Z(k)\} \neq H(k)X_2(k)$$

N equal to length of convolution;

$$z[n] = x_1[n] * h(n) + j x_2(n) * h(n)$$

$$\mathrm{Re}\{z[n]\} = x_1[n] * h(n) = y_1[n]$$

$$\mathrm{Im}\{z[n]\} = x_2[n] * h[n] = y_2(n)$$

Prob. 3 Solution

DTFT of $h[n]$: $H(\omega) = \dfrac{\sin\left(\frac{6}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\frac{(6-1)}{2}\omega}$

$h[n] = u[n] - u[n-6]$

$$H(\omega) = \frac{\sin(3\omega)}{\sin\left(\frac{\omega}{2}\right)} e^{-j\frac{5}{2}\omega}$$

$$H_8(k) = H(\omega)\Big|_{\omega = \frac{2\pi k}{8} = k\frac{\pi}{4}} \qquad k = 0, 1, \ldots, 7$$

(a)
$$= \frac{\sin\left(k\frac{3\pi}{4}\right)}{\sin\left(k\frac{\pi}{8}\right)} e^{-j\frac{5\pi}{8}k}$$

(b) $x[n] = \cos\left(\frac{4\pi}{8}n\right) + 2\cos\left(\frac{2\pi}{8}(4)n\right)$

$$= \cos\left(\frac{2\pi(2)}{8}n\right) + 2\cos\left(\frac{2\pi}{8}(4)n\right)$$

$$X_8(k) = \frac{8}{2}\delta[k-2] + \frac{8}{2}\delta[k-(8-2)] + \frac{16}{2}\delta[k-4] + \frac{16}{2}\delta[k-(8-4)]$$

$$= 4\delta[k-2] + 16\delta[k-4] + 4\delta[k-6]$$

$$Y_8[k] = X_8[k]\, H_8(k)$$

$$Y_8[k] = \frac{8}{2} H_8(2)\,\delta[k-2] + \frac{8}{2} H_8^*(2)\,\delta[k-6]$$

$$+ \; 16\, H_8(4)\,\delta[k-4]$$

$$H_8(2) = \frac{\sin\left(2\,\frac{3\pi}{4}\right)}{\sin\left(2\,\frac{\pi}{8}\right)} e^{-j\frac{10\pi}{8}} = \frac{\sin\left(\frac{3}{2}\pi\right)}{\sin\left(\frac{\pi}{4}\right)} e^{-j\frac{5\pi}{4}}$$

$$H_8(4) = \frac{\sin(3\pi)}{\sin\left(\frac{\pi}{2}\right)} e^{-j\frac{5\pi}{2}} = 0$$

$$H_8(2) = 1-j = \sqrt{2}\, e^{-j\frac{\pi}{4}}$$

$$y_8[n] = \sqrt{2}\,\cos\left(\frac{\pi}{2}n - \underbrace{45^\circ}_{\frac{\pi}{4}}\right)$$

more details:

$$\sqrt{2}\,\frac{8}{2}\, e^{-j\frac{\pi}{4}}\,\delta(k-2) + \frac{8}{2}\sqrt{2}\, e^{+j\frac{\pi}{4}}\,\delta[k-6]$$

time-domain

$$\sqrt{2}\,(2)\,e^{-j\frac{\pi}{4}}\, e^{j\,\overset{(2)}{\cancel{k}}\,\frac{2\pi}{8}n} + \sqrt{2}\,(2)\,e^{j\frac{\pi}{4}}\, e^{+j(6)\frac{2\pi}{8}n}$$

$$= e^{j(6-8)\frac{2\pi}{8}n}$$

$$\sqrt{2}\,(2)\,e^{j\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)} + \sqrt{2}\,(2)\,e^{-j\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)}$$

$$= \sqrt{2}\,\cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)$$