ECE 538 Exam 3 Sol'n Fall 2005

Problem 1. We know from development of radix 2 decimation-in-time FFT

$$X_8(k) = F_0(k) + W_8^k F_1(k)$$

$$X_8(k+4) = F_0(k) - W_8^k F_1(k)$$

$k=0,1,2,3$

$k=0$:

$$X_8(0) = 0 + 0 = 0$$

$$X_8(4) = 0 + (1)\cdot 0 = 0$$

note:

$$\boxed{\begin{aligned} W_8^3 &= e^{-j\frac{2\pi}{8}3} \\ &= e^{-j\frac{3\pi}{4}} \end{aligned}}$$

$k=1$:

$$X_8(1) = 0 + 0 = 0$$

$$X_8(5) = 0 + \frac{1}{\sqrt{2}}(1-j)\cdot 0 = 0$$

$k=2$: $X_8(2) = 1 + (-j)(-j) = 0$

$$X_8(6) = 1 - (-j)(-j) = 2$$

$k=3$: $X_8(3) = 1 + \left(-[1+j][-1+j]\right)\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 2$

$$X_8(7) = 1 - 1 = 0$$

(a)

$$X_8(k) = 2\,\delta(k-3) + 2\,\delta(k-6)$$

(b) $x[n] = \frac{1}{4}e^{j2\pi\frac{3}{8}} + \frac{1}{4}e^{j2\pi\frac{6}{8}}$

# Problem 2 Sol'n.

Given that the 7 pt DFT of $x[n]$ is <u>not</u> a simple expression, <u>and</u> that $h[n]$ is only 2 taps, easier to do <u>either</u> circular convolution or use time-domain aliasing formula $\Rightarrow$ we'll use the latter

$$y[n] = x[n] * h[n]$$

$\Rightarrow$ "differencing" filter applied to triangle fn.

$$= \{1, 1, 1, 1, -1, -1, -1, -1\}$$ (think slope!)

$\Rightarrow$ length $= 7 + 2 - 1 = 8$

• since convolution is of length 8 but we computed 7 pt DFT's, only one pt is aliased:

$$y_7[n] = y[n] + y[n+7] \qquad n = 0, 1, \ldots, 6$$

$$y_7[0] = y[0] + y[7] = 1 + (-1) = 0$$

$\Rightarrow$ last point is aliased into first pt.

$\Rightarrow$ all other points OK

$y_7[n] = y[n]$ for $n = 1, 2, 3, 4, 5, 6$

since $y[n+7] = 0$ for $n$ in that range

Answer:

$$y_7[n] = \{0, 1, 1, 1, -1, -1, -1\}$$