EE538 DSP I Sol'n to Exam 3 F 2000

Sol'n to Prob. 1

$$\mathcal{E}^0_{min} = r_{xx}[0] = 4$$

(a) $$a_1(1) = -\frac{r_{xx}[1]}{r_{xx}[0]} = -\frac{2}{4} = \frac{-1}{2}$$

$$\mathcal{E}^1_{min} = \mathcal{E}^0_{min}\left(1 - a_1^2(1)\right) = 4\left(1 - \frac{1}{4}\right) = 3$$

(b) $$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = -\begin{bmatrix} 2 \\ 1/2 \end{bmatrix}$$

$$a_2(1) = \frac{\begin{vmatrix} -2 & 2 \\ -1/2 & 4 \end{vmatrix}}{\begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix}} = \frac{-8+1}{12} = \frac{-7}{12}$$

$$a_2(2) = \frac{\begin{vmatrix} 4 & -2 \\ 2 & -1/2 \end{vmatrix}}{\begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix}} = \frac{-2+4}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\mathcal{E}^2_{min} = \mathcal{E}^1_{min}\left(1 - a_2^2(2)\right)$$

$$= 3\left(1 - \frac{1}{36}\right) = 3\,\frac{35}{36} = \frac{35}{12}$$

(c) Since $x[n]$ is an AR(2) process,

$$a_1 = a_2(1) = \frac{-7}{12}, \quad a_2 = a_2(2) = \frac{1}{6}, \quad \sigma_w^2 = \underbrace{\mathcal{E}^2_{min}}_{\text{min prediction error for any order}} = \frac{35}{12}$$

Thus:

$$a_3(1) = a_2(1) = -\frac{7}{12}$$

$$a_3(2) = a_2(2) = 1/6$$

$$a_3(3) = 0 \qquad \mathcal{E}^{min}_3 = \mathcal{E}^{min}_2 = \sigma_w^2 = \frac{35}{12}$$

Sol'n. to Prob. 1

(d) see answer to (c)

(e) $r_{xx}[3] = -a_1 r_{xx}[2] - a_2 r_{xx}[1]$

$$= \frac{7}{12}\left(\frac{1}{2}\right) - \frac{1}{6}(2) = \frac{7-8}{24} = \frac{-1}{24}$$

(f) $$S_{xx}(\omega) = \frac{\sigma_w^2}{|1 + a_1 e^{-j\omega} + a_2 e^{-j2\omega}|^2}$$

$$= \frac{35/12}{|1 - \frac{7}{12} e^{-j\omega} + \frac{1}{6} e^{-j2\omega}|^2}$$

Sol'n. to Prob. 2

$$\mathcal{E}_0^{min} = r_{xx}[0] = \sqrt{2}$$

(a) $$a_1(1) = \frac{-r_{xx}[1]}{r_{xx}[0]} = -\frac{1}{\sqrt{2}}$$

$$\mathcal{E}_1^{min} = \sqrt{2}\left(1 - \frac{1}{2}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

(b) $$\begin{bmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{bmatrix} \begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$a_2(1) = \frac{\begin{vmatrix} -1 & 1 \\ 0 & \sqrt{2} \end{vmatrix}}{\begin{vmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{vmatrix}} = \frac{-\sqrt{2}}{1} = -\sqrt{2}$$

$$a_2(2) = \frac{\begin{vmatrix} \sqrt{2} & -1 \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{vmatrix}} = 1$$

$$\mathcal{E}_2^{min} = \mathcal{E}_1^{min}(1-1) = 0$$ } perfect prediction!!

(c) Since 2nd order prediction is perfect,

$$a_3(1) = a_2(1) = -\sqrt{2}$$

$$a_3(2) = a_2(2) = 1$$

$$a_3(3) = 0$$

$$\mathcal{E}_3^{min} = \mathcal{E}_2^{min} = 0$$

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Sol'n. to Prob. 2 (cont.)

(d) Roots of $z^2 + a_2(1) z + a_2(2)$

$$= z^2 - \sqrt{2}\, z + 1$$

$$= \left(z - e^{j\frac{\pi}{4}}\right)\left(z - e^{-j\frac{\pi}{4}}\right)$$

$$= z^2 - \underbrace{2\cos\left(\tfrac{\pi}{4}\right)}_{\frac{2}{\sqrt{2}} = \sqrt{2}} z + 1$$

$$e^{j\omega_0} = e^{j\frac{\pi}{4}}$$

$$e^{-j\omega_0} = e^{-j\frac{\pi}{4}}$$

$$\Rightarrow \omega_0 = \frac{\pi}{4}$$

(e) $r_{xx}[3] = -a_2(1)\, r_{xx}[2] - a_2(2)\, r_{xx}[1]$

$$= \sqrt{2}\,(0) - 1(1) = -1$$

$$r_{xx}[3] = -1$$

(f) $r_{xx}[m] = \frac{A^2}{2}\cos(\omega_0 m)$

$$= \frac{A^2}{2}\cos\left(\frac{\pi}{4}m\right)$$

$$= \sqrt{2}\cos\left(\frac{\pi}{4}m\right)$$

$$\frac{A^2}{2} = \sqrt{2}$$

$$S_{xx}(\omega) = \sqrt{2}\,\pi\left\{\delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right)\right\}$$