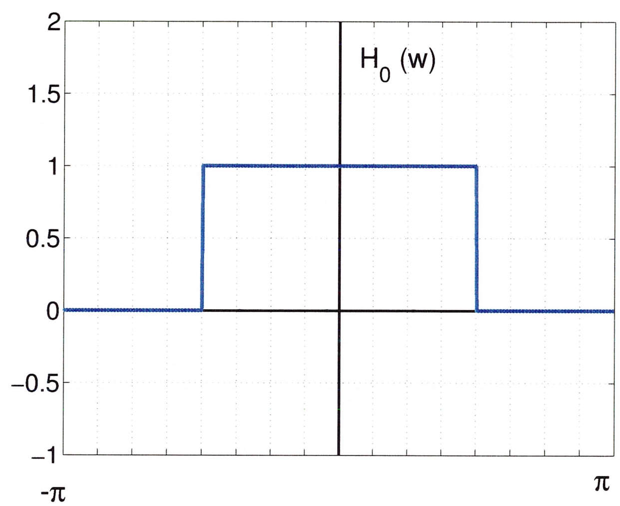
Problem 2. In Matlab Homework 2, you used Noble's Identities to convert the Tree-Structured subbander on the previous page to the regular maximally decimated subbander, obtaining products of the form:

$$H_k(\omega) = H_\ell^{(2)}(\omega) H_m^{(2)}(2\omega) H_n^{(2)}(4\omega) \quad k = 0, 1, ..., 7$$

where $\ell \in \{0, 1\}$, $m \in \{0, 1\}$, and $n \in \{0, 1\}$, with $H_0^{(2)}(\omega)$ denoting the lowpass halfband filter below for the two-channel QMF and $H_1^{(2)}(\omega) = H_0^{(2)}(\omega - \pi)$. For this problem, $H_0^{(2)}(\omega)$ is the ideal lowpass filter below (note: a DTFT is always periodic with period 2π.)



You are required to fill in the table below. **FIRST** you should plot $H_0^{(2)}(2\omega)$ and $H_0^{(2)}(4\omega)$ on the next page, and then plot $H_1^{(2)}(\omega)$, $H_1^{(2)}(2\omega)$ and $H_1^{(2)}(4\omega)$ on the next page after that, in the space provided. In each plot, the abscissa range is from $-\pi$ to $+\pi$ and there is a tic mark and vertical dashed line at every integer multiple of $\pi/8$. These plots will help you fill in the table. Only fill in the positive frequency band that is passed; the filters are real-valued and even-symmetric, so their respective frequency responses are real-valued and even-symmetric.

$H_1^{(2)}(\omega)H_0^{(2)}(2\omega)H_0^{(2)}(4\omega)$ passes: $7\pi/8 \leq \omega < \pi$

$H_1^{(2)}(\omega)H_0^{(2)}(2\omega)H_1^{(2)}(4\omega)$ passes: $6\pi/8 < \omega < 7\pi/8$

$H_1^{(2)}(\omega)H_1^{(2)}(2\omega)H_1^{(2)}(4\omega)$ passes: $5\pi/8 < \omega < 6\pi/8$

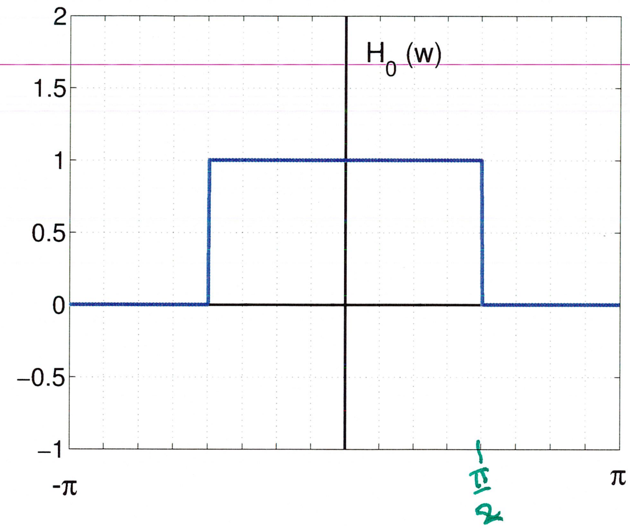
$H_1^{(2)}(\omega)H_1^{(2)}(2\omega)H_0^{(2)}(4\omega)$ passes: $4\pi/8 < \omega < 5\pi/8$

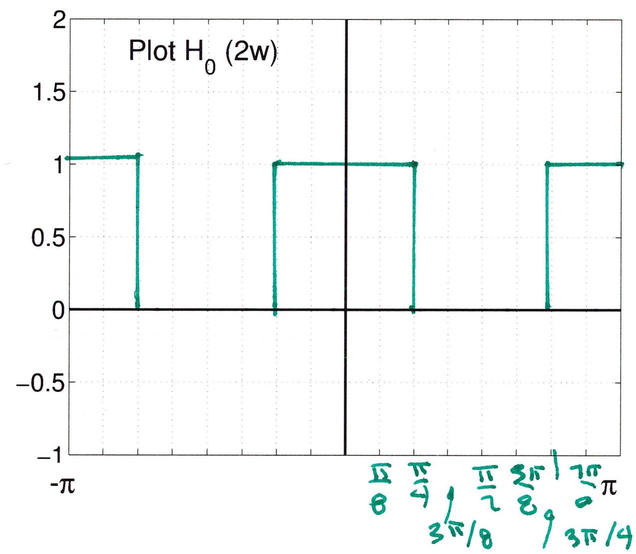
$H_0^{(2)}(\omega)H_1^{(2)}(2\omega)H_0^{(2)}(4\omega)$ passes: $3\pi/8 < \omega < 4\pi/8$ (1)

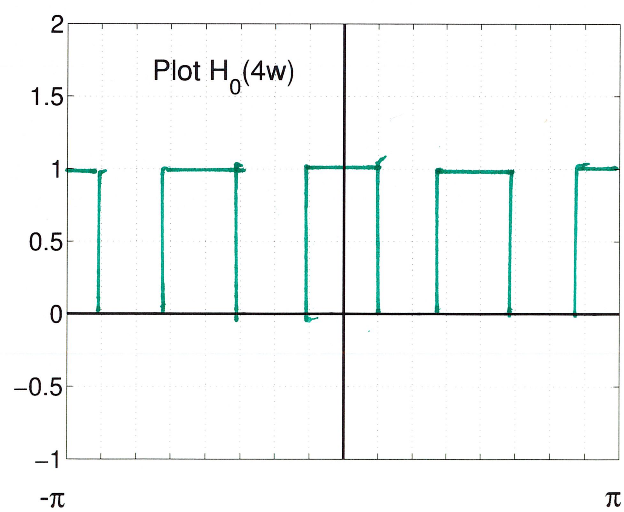
$H_0^{(2)}(\omega)H_1^{(2)}(2\omega)H_1^{(2)}(4\omega)$ passes: $2\pi/8 < \omega < 3\pi/8$

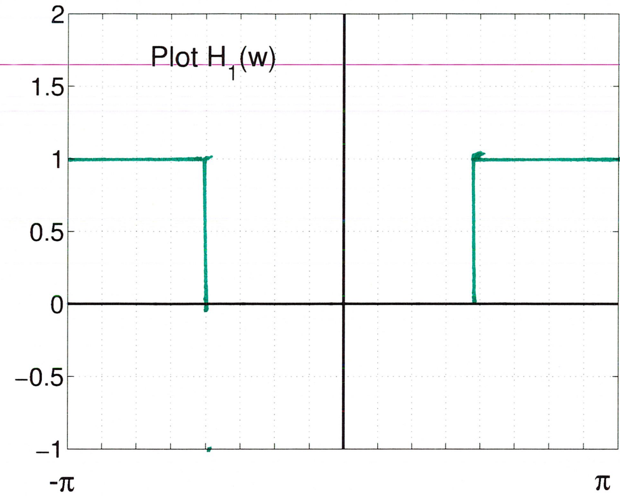
$H_0^{(2)}(\omega)H_0^{(2)}(2\omega)H_1^{(2)}(4\omega)$ passes: $\pi/8 < \omega < 2\pi/8$

$H_0^{(2)}(\omega)H_0^{(2)}(2\omega)H_0^{(2)}(4\omega)$ passes: $0 < \omega < \pi/8$

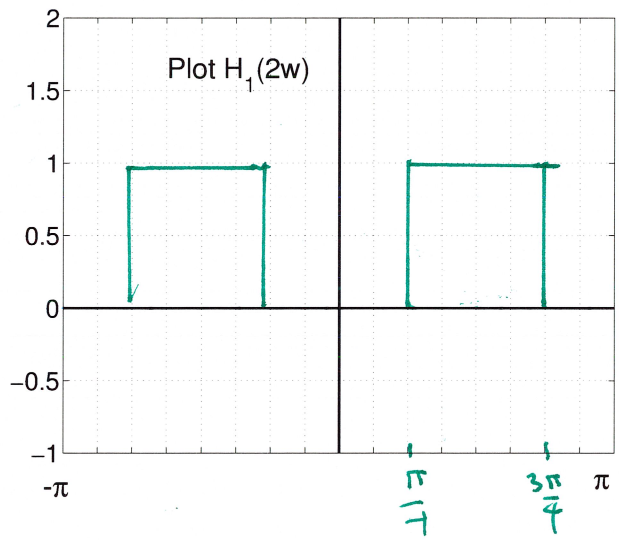

H0 (w)
2
1.5
1
0.5
0
−0.5
−1
-π
π
π/2


Plot H0 (2w)
2
1.5
1
0.5
0
−0.5
−1
-π
π
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8

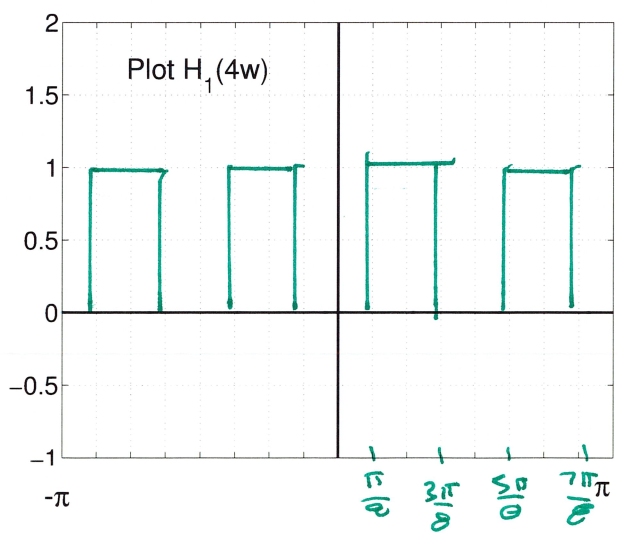

Plot H0(4w)
2
1.5
1
0.5
0
−0.5
−1
-π
π



Plot $H_1(w)$
2
1.5
1
0.5
0
−0.5
−1
-π
π



Plot $H_1(2w)$
2
1.5
1
0.5
0
−0.5
−1
-π
π/4
3π/4
π



Plot $H_1(4w)$
2
1.5
1
0.5
0
−0.5
−1
-π
π/8
3π/8
5π/8
7π/8
π