ECE538 **Exam 3 Take-Home** **Fall 2024**
Digital Signal Processing I **Due: 1 December 2024**

Cover Sheet

Test Duration: Due uploaded to Brightspace by 11:59 EST, Dec. 1 .
This test contains **three** problems.
You must show ALL work for each problem to receive full credit.

No.	**Topic(s) of Problem**	**Points**
1.	OFDM	25
2.	Noble's Identities	25
3.	Perfect Reconstruction Filter Bank	50

Problem 1. An OFDM signal is synthesized as a sum of four sinewaves of length 4 as

$$\tilde{x}[n] = \frac{1}{4}\sum_{k=0}^{3} b_k e^{j2\pi\frac{k}{4}n}\{u[n] - u[n-4]\}$$

Each of the four values b_k is one of the four QPSK values listed in the symbol alphabet below.

$$b_k \in \{1+j, 1-j, -1+j, -1-j\} \quad k = 0, 1, 2, 3$$

We know in advance that the signal we transmit will be convolved with a filter of length $L = 3$. Thus, we add a cyclic prefix of length 3 which effective creates a sum of sinewaves of length 7 as prescribed below (note the division by 4)

$$x[n] = \frac{1}{4}\sum_{k=0}^{3} b_k e^{j2\pi\frac{k}{4}n}\{u[n+3] - u[n-4]\}$$

The sequence of length 7 above is convolved with the filter below of length 3

$$h[n] = \{8, 4, 2\} = 8\delta[n] + 4\delta[n-1] + 2\delta[n-2]$$

This ultimately yields the following sequence of length 7+3-1 = 9

$$y[n] = x[n] * h[n] = \{4+4j, -2+6j, 3+7j, 5-j, 7+3j, -1+5j, 3+7j, 1+3j, 1+j\}$$

Your task is to determine the numerical values of each of the four symbols: b_0, b_1, b_2, and b_3. Explain and show all the steps in determining your answers. Please lay your work out nicely with logical ordering. Your work will be more important than your final answers.

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Problem 2. Determine and plot the frequency response $H_7(\omega)$ in the bottom chain in Figure 2(a) below so that the I/O relationship at the bottom in Figure 2(b) (next page) is exactly the same as the I/O relationship of the bottom chain in Figure 2(a) below, given the ideal $h_0^{(2)}[n]$ and $h_1^{(2)}[n]$ defined below. Plot the magnitude of the DTFT $H_7(\omega)$ over $-\pi < \omega < \pi$. Show all work. In particular, include plots of (i) $|H_0^{(2)}(\omega)|$, (ii) $|H_0^{(2)}(2\omega)|$, (iii) $|H_0^{(2)}(4\omega)|$, (iv) $|H_1^{(2)}(\omega)|$, (v) $|H_1^{(2)}(2\omega)|$, (vi) $|H_1^{(2)}(4\omega)|$.

$$h_0^{(2)}[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \qquad\qquad h_1^{(2)}[n] = e^{j\pi n}\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$$

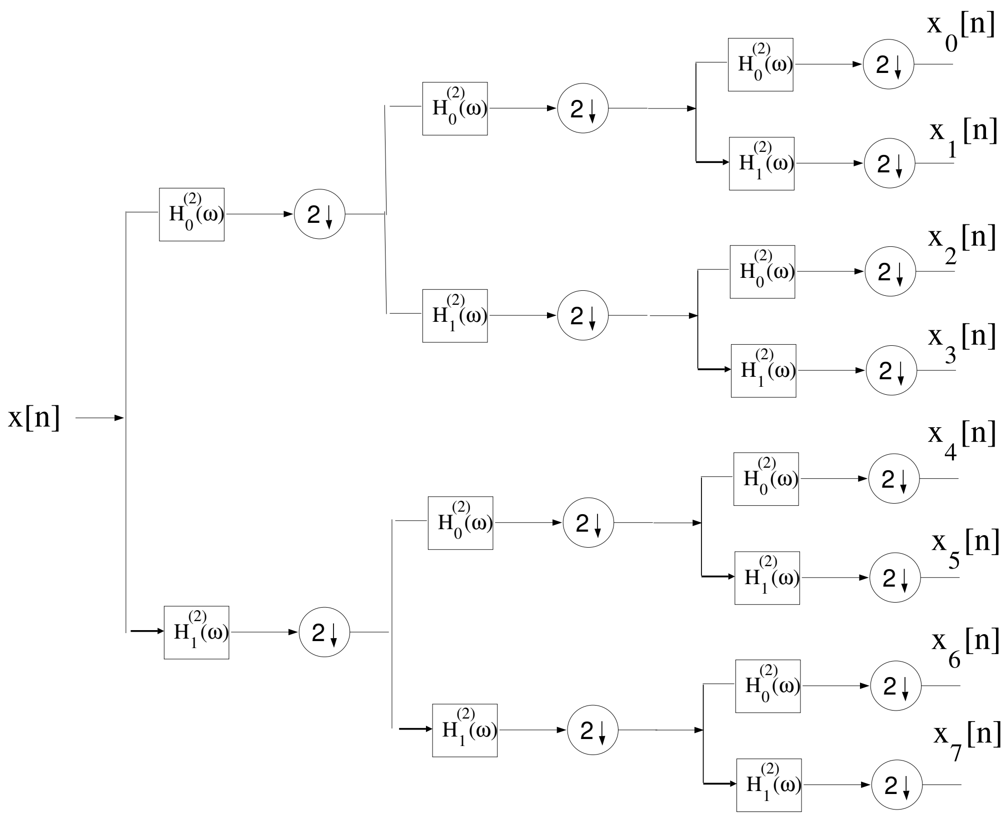

Figure 2(a). Analysis Section of Three-Stage Tree-Structured Filter Bank.

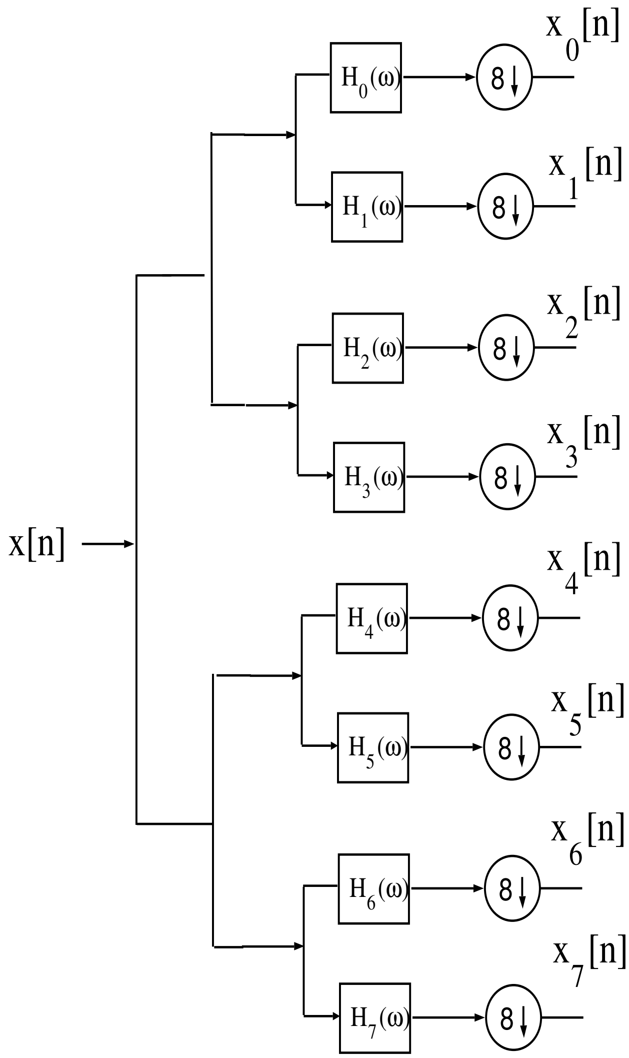

Figure 2(b). Analysis Filter Bank, $M = 8$.

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Problem 3. Consider the $M = 4$ channel Filter Bank in Figure 3.1 on the next page. Review the derivation of the 2-channel QMF-based PRFB course notes posted at the course web site (https://engineering.purdue.edu/ ee538/TwoChannelQMF.pdf) You are to generalize that derivation for the case of 4 channels (Fig. 3.1) but for the specific case where the four real-valued filters are of length 4 as defined below.

(i) Using (https://engineering.purdue.edu/ ee538/VIP_MultirateFormulas.pdf) VIP Multirate Formulas, develop the matrix set of equations $\mathbf{V} = \mathbf{H}\mathbf{X}$ where $\mathbf{V} = [V_0(\omega), V_1(\omega), V_2(\omega), V_3(\omega)]^T$ is 4x1, $\mathbf{H}$ is 4x4 and contains $H_k(\omega)$, k=0,1,2,3, and frequency shifted versions of $H_k(\omega)$ as entries, and $\mathbf{X}$ is 4x1 containing $X(\omega)$ and frequency shifted versions of $X(\omega)$ as entries. Show all work: Show all work: this is your chance to show you know to apply all the Multirate Formulas derived in class.

(ii) Determine whether Perfect Reconstruction is achieved with the four **causal** length-4 analysis filters listed below and the corresponding four synthesis filters $g_k[n] = h_k[-n], k = 0, 1, 2, 3$. Show all work. **Note:** achieving Perfect Reconstruction means the output is the same as the input except for possibly an amplitude-scaling and an integer-delay (time-shift.)

$$h_0[n] = \{1, 2, 3, 4\}$$

$$h_1[n] = \{-2, 1, -4, 3\}$$

$$h_2[n] = \{-3, 4, 1, -2\}$$

$$h_3[n] = \{-4, -3, 2, 1\}$$

(iii) Determine and draw a computationally efficient implementation for the analysis side, using the four analysis filters defined in part (ii.)

(iv) Determine and draw a computationally efficient implementation for the synthesis side, four synthesis filters $g_k[n] = h_k[-n], k = 0, 1, 2, 3$, where $h_k[n]$ are the four analysis filters defined in part (ii.)

Note, the four analysis filters and the four synthesis filters are all real-valued and EACH is of length 4. Every quantity in this problem is real-valued. **SHOW ALL WORK: an answer with no supporting work will receive very little partial credit. This is a chance for you to show what you know about the efficient implementation of "filtering followed by decimation" AND the efficient implementation of "zero-inserts followed by filtering."**

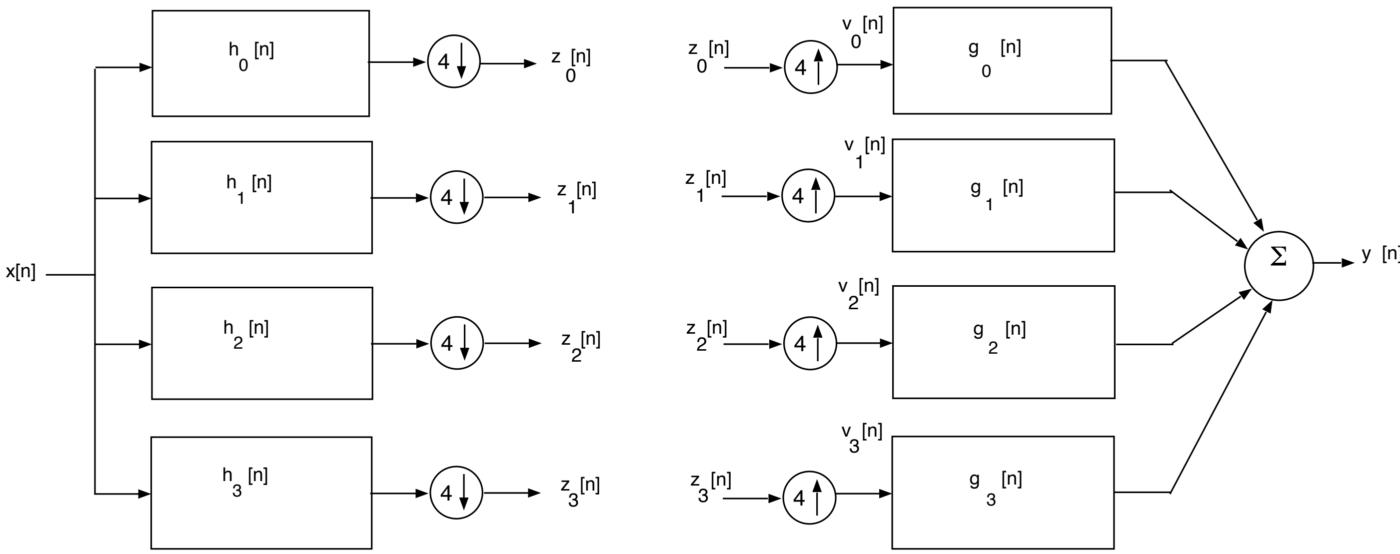

Figure 1(a). Analysis Filter Bank. Figure 1(b). Synthesis Filter Bank.

Figure 3.1. Problem 3.

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