NAME: **2018**
Digital Signal Processing I **Exam 3** **Fall 2018**
Session 40 **30 Nov. 2018**

Cover Sheet

Test Duration: 60 minutes.
Open Book but Closed Notes. One 8.5 x 11 crib sheet allowed
Calculators NOT allowed.
Show all work. More credit for approach than final answer.
This test contains **THREE** problems.
All work should be done on the blank pages provided.
Your answer to each part of the exam should be clearly labeled.

GIVEN NOBLE's IDENTITIES TO USE IN PROBLEM 1.

(a) If $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

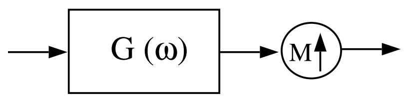

Figure 1(a).

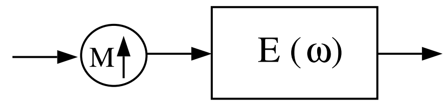

Figure 1(b).

(b) If $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) satisfies $F(\omega) = H(M\omega)$, the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

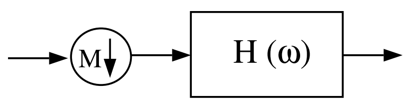

Figure 2(a).

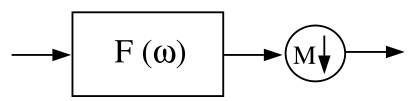

Figure 2(b).

Problem 1.

(a) Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$. *Hint:* Analyze the system of Figure 3(a) in the frequency domain using Noble's First Identity.

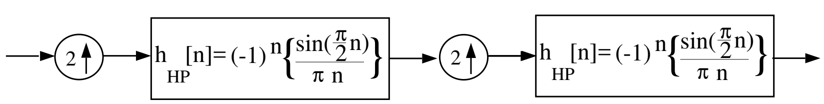

Figure 3(a).

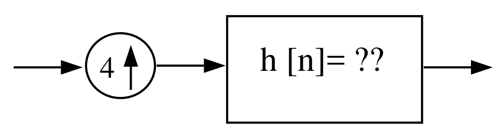

Figure 3(b).

(b) Determine the numerical values of the impulse response $h_{\text{eq}}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). *Hint:* Analyze the system of Figure 4(a) in the time domain using Noble's First Identity.

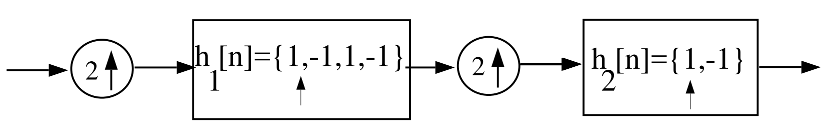

Figure 4(a).

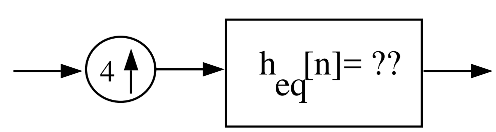

Figure 4(b).

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Problem 2. Consider a causal FIR filter of length $M = 8$ with impulse response as defined below:

$$h[n] = 2\sin\left(\frac{3\pi}{4}n\right)\{u[n] - u[n-8]\}$$

Consider a DT sinewave $x[n]$ of length $N = 16$ as defined below:

$$x[n] = \left\{\cos\left(\frac{\pi}{2}n\right) + \cos\left(\frac{12\pi}{16}n\right)\right\}\{u[n] - u[n-16]\}$$

$y[n]$ is formed as the linear convolution of $x[n]$ with $h[n]$ as:

$$y[n] = x[n] * h[n]$$

We then take the last $M - 1 = 7$ values of $y[n]$ and time-domain alias them into the first three values to form a sequence of length 16, denoted $y_a[n]$, according to:

$$y_a[n] = y[n] + y[n+16], n = 0, 1, 2, ..., 6 \quad (1)$$
$$y_a[n] = y[n], n = 7, 8, 9, 10, 11, 12, 13, 14, 15 \quad (2)$$

Determine an expression for $y_a[n]$ similar to the expression for $x[n]$ above. Show all work. You do NOT have to list the 16 numerical values of $y_a[n], n = 0, 1, ..., 15$ in sequence form. **NOTE 1:** Using concepts learned in class; There won't be a lot of points for brute force calculations that do not show what you learned in class for Exam 3.

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Problem 3. Consider a causal FIR filter of length $M = 9$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \left\{ 2\frac{\sin\left[\frac{\pi}{4}(n+\ell 9)\right]}{\pi(n+\ell 9)} - 4\frac{\sin\left[\frac{\pi}{2}(n+\ell 9)\right]}{\pi(n+\ell 9)} + 3\frac{\sin\left[\frac{3\pi}{4}(n+\ell 9)\right]}{\pi(n+\ell 9)} \right\} \{u[n] - u[n-9]\}$$

(a) Determine the 9-pt DFT of $h_p[n]$, denoted $H_9(k)$, for $0 \leq k \leq 9$. Write your answer in sequence form to indicate the numerical values of $H_9(k), k = 0, 1, ..., 8$.

(b) Consider the sequence $x[n]$ of length $L = 9$ below.

$$x[n] = \left\{ -\cos\left(\frac{4\pi}{9}n\right) + 2\sin\left(\frac{8\pi}{9}n\right) + \frac{1}{3}\cos\left(\frac{4\pi}{3}n\right) \right\} \{u[n] - u[n-9]\}$$

$y_9[n]$ is formed by computing $X_9(k)$ as a 9-pt DFT of $x[n]$, $H_9(k)$ as a 9-pt DFT of $h[n]$ and, finally, then $y_9[n]$ is computed as the 9-pt inverse DFT of $Y_9(k) = X_9(k)H_9(k)$. Express the result $y_9[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

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