**NAME: 2018**
**Digital Signal Processing I — Exam 3 — Fall 2018**
**Session 40 — 30 Nov. 2018**

## Cover Sheet

Test Duration: 60 minutes.
Open Book but Closed Notes. One 8.5 x 11 crib sheet allowed
Calculators NOT allowed.
Show all work. More credit for approach than final answer.
This test contains **THREE** problems.
All work should be done on the blank pages provided.
Your answer to each part of the exam should be clearly labeled.

**GIVEN NOBLE's IDENTITIES TO USE IN PROBLEM 1.**

(a) If $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

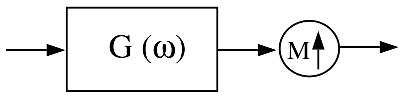

Figure 1(a).

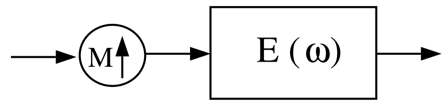

Figure 1(b).

(b) If $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) satisfies $F(\omega) = H(M\omega)$, the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

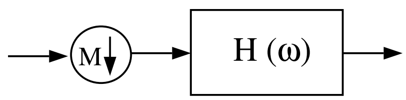

Figure 2(a).

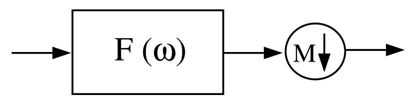

Figure 2(b).

**Problem 1.**

(a) Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$. *Hint:* Analyze the system of Figure 3(a) in the frequency domain using Noble's First Identity.

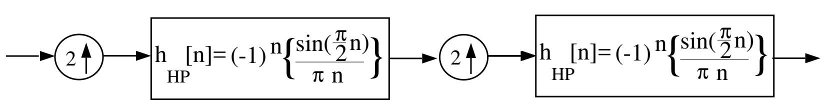

Figure 3(a).

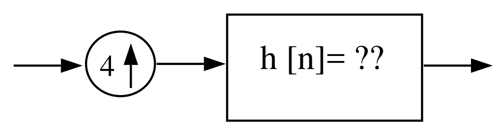

Figure 3(b).

(b) Determine the numerical values of the impulse response $h_{\mathrm{eq}}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). *Hint:* Analyze the system of Figure 4(a) in the time domain using Noble's First Identity.

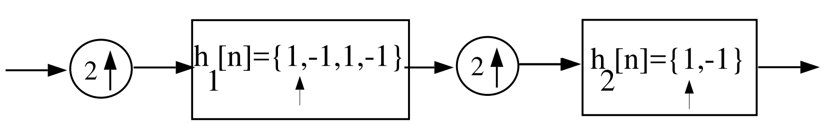

Figure 4(a).

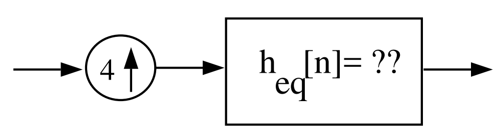

Figure 4(b).

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**Problem 2.** Consider a causal FIR filter of length $M = 8$ with impulse response as defined below:

$$h[n] = 2\sin\left(\frac{3\pi}{4}n\right)\{u[n] - u[n-8]\}$$

Consider a DT sinewave $x[n]$ of length $N = 16$ as defined below:

$$x[n] = \left\{\cos\left(\frac{\pi}{2}n\right) + \cos\left(\frac{12\pi}{16}n\right)\right\}\{u[n] - u[n-16]\}$$

$y[n]$ is formed as the linear convolution of $x[n]$ with $h[n]$ as:

$$y[n] = x[n] * h[n]$$

We then take the last $M - 1 = 7$ values of $y[n]$ and time-domain alias them into the first seven values to form a sequence of length 16, denoted $y_a[n]$, according to:

$$\begin{aligned} y_a[n] &= y[n] + y[n+16], n = 0, 1, 2, ..., 6 && (1) \\ y_a[n] &= y[n], n = 7, 8, 9, 10, 11, 12, 13, 14, 15 && (2) \end{aligned}$$

Determine an expression for $y_a[n]$ similar to the expression for $x[n]$ above. Show all work. You do NOT have to list the 16 numerical values of $y_a[n], n = 0, 1, ..., 15$ in sequence form. **NOTE 1:** Using concepts learned in class; There won't be a lot of points for brute force calculations that do not show what you learned in class for Exam 3.

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**Problem 3.** Consider a causal FIR filter of length $M = 9$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \left\{ 2\frac{\sin\left[\frac{\pi}{4}(n+\ell 9)\right]}{\pi(n+\ell 9)} - 4\frac{\sin\left[\frac{\pi}{2}(n+\ell 9)\right]}{\pi(n+\ell 9)} + 3\frac{\sin\left[\frac{3\pi}{4}(n+\ell 9)\right]}{\pi(n+\ell 9)} \right\} \{u[n] - u[n-9]\}$$

(a) Determine the 9-pt DFT of $h_p[n]$, denoted $H_9(k)$, for $0 \leq k \leq 9$. Write your answer in sequence form to indicate the numerical values of $H_9(k), k = 0, 1, ..., 8$.

(b) Consider the sequence $x[n]$ of length $L = 9$ below.

$$x[n] = \left\{ -\cos\left(\frac{4\pi}{9}n\right) + 2\sin\left(\frac{8\pi}{9}n\right) + \frac{1}{3}\cos\left(\frac{4\pi}{3}n\right) \right\} \{u[n] - u[n-9]\}$$

$y_9[n]$ is formed by computing $X_9(k)$ as a 9-pt DFT of $x[n]$, $H_9(k)$ as a 9-pt DFT of $h[n]$ and, finally, then $y_9[n]$ is computed as the 9-pt inverse DFT of $Y_9(k) = X_9(k)H_9(k)$. Express the result $y_9[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

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